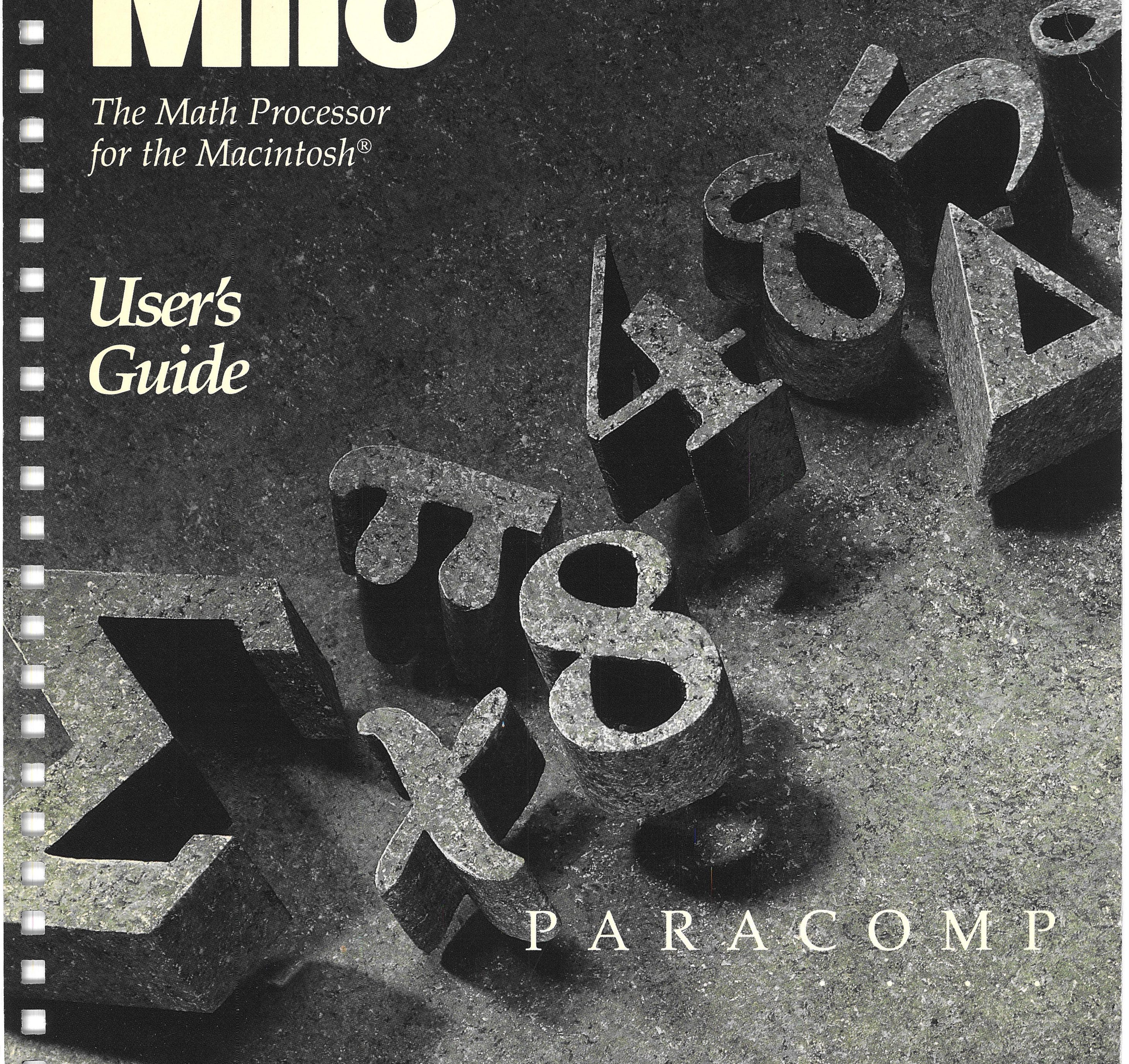
Milo™

The Math Processor for the Macintosh®

User's Guide



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Congratulations:

You have just purchased a unique leading-edge product, part of a family of products published by Paracomp in response to your needs. Milo is the first WYSIWYG math equation processor for the Mac. Milo allows you to solve mathematical equations, effortlessly graph the results and easily explain the results through its technical word processing capabilities. All this as if you were working on a normal piece of paper without having to worry about erasing. Plus, all of these features come to you without ever having to learn a very time-consuming programming language. Why? Because there is none!

Paracomp wants to bring out the best programs we can. We strive for uniqueness in both our developments and in the programs we publish. It is of extreme importance that we offer you something that will benefit your daily life. Milo is one such product. I know, for example, Milo would have made a fundamental difference to my own math experiences and to the results I received during my college math courses.

In addition to Milo, Paracomp has already released Swivel 3D, the first of a new genre of graphics-based drawing and modeling packages, which for the first time brings mainframe capabilities to the Macintosh computers of today. In the near future, expect several essential packages for your engineering and scientific needs from Paracomp. Many of these will be complimentary to Milo and bring you closer to a complete technical solution. Look for a closer relationship between Milo and your favorite math and word processing programs in the near future.

At Paracomp we listen to our customers, your suggestions are very important to us as we do aggressively upgrade our products. Our support and technical teams will be happy to take any of your calls so don't hesitate.

Many people have led to the success of Milo. I would like to thank Ron Avitzur for his commitment and for allowing us to make Milo a commercial reality. I also would like to thank Neville Campbell for his product management and Jonathan Perrow for his excellent writing of the users manual that should add immeasurably to your understanding of Milo. Finally, I would like to thank the entire Paracomp team for all their hard work.

I believe Milo is be the electronic scratchpad of choice for anyone using mathematics.

My best,

Bill Woodward
November 1988

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Acknowledgements

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Finally, thanks to my parents who have supported and trusted me.

Ron Avitzur

November 1988

Milo ™

The Software:

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Technical Consultants: Steve Roy, Mark Armstrong, and Bruce Leak
Interface Design: Jim Phillips, Steve Roy, Andrew Gooding, Mike Seiffert, and Mark Carmichael

User's Guide credits:

Writing by Jonathan Perrow
Editing by Jonathan Perrow, Neville Campbell, Paul Rosenfeld, Bill Woodward, Sean McKenna, and Ron Avitzur
Design by Rollinson Design
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For assistance with the software:

This user's guide is designed to help with most questions about Milo. Should questions arise that are not answered here, please call Paracomp Tech Support at (415) 543-3848 between 9:00 a.m. and 5:00 p.m., Pacific Coast Time.
Paracomp, Inc. is located at 123 Townsend St., Suite 310, San Francisco, CA. 94107.

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Milo™

The Math Processor
for the Macintosh®

User's Guide

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Introduction

Milo is the first intelligent piece of paper for mathematics: a fast, easy to use, powerful and intuitive math processor for the Macintosh. Milo lets you enter, manipulate, simplify and graph mathematical equations and text in a completely what-you-see-is-what-you-get (WYSIWYG) manner unlike any math program available for the Mac. To use Milo, you must have an understanding of the Macintosh and also of mathematics. Just as a word processor does not "know" English, Milo does not "know" math. Milo will be most useful when you already know the steps that lead to solving your problem. If you supply the understanding and can lead Milo through each step of a problem, it will keep track of the minus signs and factors of two and work through all the steps quickly. Milo lets you try out different ideas in the language of mathematics and provides instant visual results.

What you need to use Milo

Milo will run on any Apple Macintosh computer with 512 KBytes or more of memory and an internal double-sided disk drive. That is, it will run on a Mac 512KE but not on a 128K or an old 512K machine with a single-sided drive that still has the 64K ROMs. If you have only 512 KBytes of memory, it will be difficult to work with large documents or to have very many documents open at once. Milo will run comfortably on a Macintosh Plus or SE, and, of course, on a Mac II. Under MultiFinder it will run under a 384K partition, but a larger partition is recommended, when working with large documents or multiple Milo files.

About system software

Milo will not work with System 4.0 or earlier. It requires System 4.1 or later. Due to space limitations, System 4.2 is included with Milo; it contains few fonts and desk accessories and does not contain the LaserWriter resources.

The Milo application is in a file on the Milo distribution disk you have received. The disk is not copy protected. The next chapter explains how to copy the disk, if you are not sure of the procedure. Use the copy and put the original Milo disk in a safe place in case the copy is damaged. You should carefully read and understand Paracomp's licensing agreement before you proceed.

About this user's guide

This user's guide describes what Milo can do and shows you how to put the program to work. You should already be familiar with the Macintosh, at least the basics of pulling down menus, selecting, clicking and dragging with the mouse.

If you are new to the Mac, now is a good time to whip out a Macintosh manual and get up to speed before reading any more about Milo.

This user's guide is divided into two sections, the Tutorial and Overview section and the Reference section. Chapters one through three constitute the Tutorial and Overview section, offering a "hands-on" approach to learning the program. Chapters four through nine make up the Reference section in which commands and other aspects of the program are explained in detail.

This chapter covers what you will need to use Milo, and what you should expect to get out of the program and this user's guide.

Tutorial Section

Chapter 1, "Overview" lays the groundwork, giving you a general understanding of how Milo works and how to use it to write, simplify and solve simple equations.

Chapter 2, "Getting Started" helps you get Milo up and running on your Macintosh.

Chapter 3, "Tutorials" offers an overview of the program through five tutorials that contain examples of Milo in action. The tutorials offer step-by-step instructions on how to use Milo to solve specific problems from graphing transcendental systems of equations to finding the area bounded by a curve.

Reference Section

Chapter 4, "Entering Expressions" provides a complete reference to the many and useful ways that mathematical expressions can be entered into Milo.

Chapter 5, "Rewriting Expressions" explains all the ways that Milo manipulates and simplifies expressions you have already entered.

Chapter 6, "Using Rules" documents the commands for customizing Milo's built-in pattern matcher to simplify expressions.

Chapter 7, "Graphing Expressions" shows you how to graph equations using Milo's powerful graphing interface.

Chapter 8, "Other Features of Milo" demonstrates some of the options available in the program.

Chapter 9, "Tips and Techniques" lists many of the shortcuts and techniques that will help you use Milo efficiently and includes a quick-reference guide to Milo commands.

Bulletin Board

Paracomp, Inc. provides a bulletin board for registered owners of their products at (415) 882-0523. We encourage users to upload any documents they would like to make publicly available to demonstrate unique uses of their products. In the case of Milo, this is an excellent medium to build up large integral tables and other math tables.

Also, the bulletin board can be used to ask questions and solve common problems. Bug reports and suggestions for future improvements may be made directly to Paracomp.

Newsletter

Registered users will receive a special Milo newsletter direct from Paracomp, Inc. to suggest new applications and answer common questions.

Hotline

For help solving problems and assistance with technical questions please call Paracomp Technical Support at (415) 543-3848 between 9:00 a.m. and 5:00 p.m. Western time.

Notation and abbreviations

To avoid confusion, the following special notation is used in certain places:

$\boxed{\text{A}}$ A box drawn around a letter, number or other symbol is used to indicate keyboard input into Milo. The keys $\boxed{⌘}$, $\boxed{\text{opt}}$ and $\boxed{\text{shift}}$ are followed by a dash and another boxed symbol like this: $\boxed{⌘}$ - $\boxed{\text{A}}$. This indicates that the command key should be held down while typing an "A."

Bold To indicate noteworthy features of Milo that may be different from what you might expect.

Keyboard input is indicated by the boxed symbols described above. In certain instances, this notation does not show the shift key being held down. For example, if you are entering expressions from the keyboard, some mathematical operators will appear differently in this mode than they do when you see them on the screen. For example, typing $\boxed{3}$ $\boxed{\text{shift}}$-$\boxed{6}$ $\boxed{2}$ in Milo will produce 3^2. For clarity, the keyboard combination $\boxed{\text{shift}}$-$\boxed{6}$ will be signified as $\boxed{\wedge}$ throughout this text.

The disk that comes with this user's guide will be referred to as the Milo distribution disk.

Special keys

The following keys perform special functions in Milo:

Key	Notation	Function
Command	$\boxed{\text{⌘}}$	Used as a control key to alter the meaning of characters typed at the keyboard.
Option	$\boxed{\text{opt}}$	Used in the same way as the command key
Space	$\boxed{\text{sp}}$	Expands selected range of an expression.
Left and Right Arrow	$\boxed{\leftarrow}$, $\boxed{\rightarrow}$	Traverse the expression tree.
Up and Down Arrow	$\boxed{\uparrow}$, $\boxed{\downarrow}$	Increment or decrement selected range.
Tab	$\boxed{\text{tab}}$	Cycles the insertion point through the prompts in a template.
Return	$\boxed{\text{return}}$	Creates a new line in an equation block, or a new row in a matrix, depending on the context.
Enter	$\boxed{\text{enter}}$	Switches Milo from Text Block mode to Equation block mode.
Comma	$\boxed{,}$	Creates a new column in a matrix, adds a new parameter to a function or creates upper or lower index, depending on the context.

Period	.	Puts a dot over or creates a string "0."
Backslash	\	Defines strings for runtime macro definitions.
Hat	^	Exponential Operator, short for shift-6

Character differences

Throughout the user's guide, the upper case letter "i" is referred to as: I, or I
The digit 1 will look like this: 1
The capital letter O will look like this: O
The digit zero, when typed from the keyboard will look like this: 0

Expressions, subexpressions and equations

This user's guide makes frequent reference to the words "expression," "subexpression" and "equation." Milo is a symbolic math program that manipulates mathematical *expressions*. Expressions can be as simple as the variable "x," or as complicated as your imagination and the program together will allow. Expressions that include the equals sign operator are called *equations* and expressions that can be broken down into smaller expressions are said to be made up of *subexpressions*.

The Milo distribution disk

The Milo distribution disk is an 800K-Byte double-sided disk that contains everything you need to use Milo:

- Milo: The application has an **M** icon
- Backslash.ini: a Milo file containing macro definitions loaded at startup
- System Folder: folder containing System 6.0, Finder 6.0
- Tutorials: folder containing tutorials for Milo
- Tables of Formulas: folder containing tables of mathematical formulas

Chapter 1 Overview

What this chapter contains

Even if you consider yourself an experienced Macintosh user, you should read this chapter. For one thing, it is short. For another, it introduces the features of Milo. The discussion that follows explains how Milo works and outlines the two basic stages of the program: the input stage and the rewrite stage.

- Inputting text
- Inputting graphics
- Inputting equations
- Selecting expressions
- Rewriting expressions

Why you should read this chapter

If you don't know what Milo can do, you won't be able to use it.

Text, graphics and equations

Milo is a powerful symbolic math processor. There are, however, three types of "blocks" that Milo can manipulate in a document: text, graphics and equations. Most of the power of Milo lies in the equation block where expressions are input, manipulated, and rewritten. Each block may contain only one type of input, but you can have as many blocks as you like in a document. This allows you to combine the different elements smoothly and easily in one document.

If the insertion point is in an equation block, the menu bar will display menus for entering and manipulating equations. If the insertion point is in a text block, text-editing menus will be displayed and for a graphics block the menus are altered accordingly.

Inputting text

In a text block, you can input text using Milo's built-in word processor. You can select, delete, cut, copy, paste, size and style in a standard Macintosh way. Since the text editor was designed primarily for writing comments between equations, **Milo does not handle text blocks larger than 32,767 characters, or about 4,000 words**. This means you should not try to squeeze Moby Dick into a text block, but for most applications an average of over 4,000 words is more than enough. Text may be cut and pasted to and from Macintosh word processors and other text processing packages.

Inputting graphics

Milo graphs functions in a separate graphing window. From that window, you can paste a graph into the clipboard and from there directly into a document window. The "Graph Expression" command in the Edit menu starts the graphing procedure once an expression has been selected and lets you configure the horizontal and vertical coordinates of the graph. From a graphics block, typing enter will put the program in text block mode, and typing return will put the program in equation block mode. Any kind of graphics can be pasted into a graphics block of the document window from the Scrapbook.

Inputting expressions

Equation block mode is the most powerful and most used mode of Milo. In the input mode of an equation block, Milo should be thought of as a math processor, similar in function to a word processor or a food processor, except for math. There are two menus full of mathematical templates, a menu for diacritical markings, and a range of keyboard input options that let you build highly complicated mathematical expressions in an equation block. Before you do anything mathematical with Milo, you have to input an expression. When you select a template or diacritical mark from the menu, Milo automatically inserts your selection into the equation block. Typically, one expression can be input per line, and each line extends horizontally up to 32,000 pixels, over 60 screen widths.

From an equation block, typing the enter key will put the program in a text block, and typing the return key will create a new equation block.

Selecting expressions

In keeping with Charles Darwin, the selection process in Milo is quite natural once you get the hang of it. In order to input and rewrite an expression you will need to be comfortable with selecting expressions and parts of expressions. When you select an expression, you prepare it to be operated on by a rewrite rule, an operator, or Milo's Graph Expression command. A selected expression or subexpression is highlighted in white typeface with a black background. You can select any expression or subexpression by either clicking with the mouse, using the space bar, or using the arrow keys. Milo will let you select only valid subexpressions.

Runtime definitions

The Milo distribution disk includes a library of runtime definitions to make the input stage even easier. These are also referred to as standard macros, and they are found in a file called "Backslash.ini." Holding down the caps lock key when you boot the program tells Milo not to load the runtime library and speeds up the initial loading process. This means, however, that the runtime definitions will not be available.

Typing a backslash \ followed by the name of a mathematical operation and then a space sp tells Milo to search through the runtime definitions library for a template corresponding to the name you entered. The space bar selects what you have typed and the backslash key tells Milo to check through the library. If it finds a match, Milo will insert the template in place of the word. For example, typing \ S U M sp tells Milo to search through the runtime library and insert the template:

$$\sum_{?=?}^{?} ?$$

This offers an alternative to choosing the summation template from the Templates menu.

You can browse through the Backslash.ini file on the Milo distribution disk by using the Open command in the file menu while running Milo.

Expression syntax in Milo

All expressions in Milo are built from objects. Here are some examples of each kind of object in Milo:

Atoms (Keyboard)		
	Numbers	3, 8, 10, 50
	Letters	a, b, c, a, b, g
	Strings	mass , meters_per_second
	Operators	+, -, /
Diacritical Marks (Diac menu)		$\hat{a}$, $\tilde{\underline{a}}$, a'
Templates (Templates menu)		$\sqrt{?}$, $\int ? d?$, *sin* ?

Strings as atoms

Atoms can be numbers, letters or symbols typed in from the keyboard. When you type letters one after the other, they are considered by Milo to be individual atoms multiplied together. To create single variables longer than one letter, you use a string. Strings are invoked by typing the single quote key ' or the backslash key \.

At first, strings can look the same as several atoms multiplied together. To tell the difference, look at the space between the characters. Strings are tightly packed sequences of characters, while atoms are spaced farther apart.

This is a string, considered as one atom:

centimeter

This is 10 atoms multiplied together:

c e n t i m e t e r

If you use Smart Quotes or any other program that converts straight quotes into curly quotes, it must be disabled in order for Milo to create strings in that manner.

As you can see, mathematical objects in Milo can be *atoms* or *templates*. Atoms are the simplest objects from which Milo expressions are built and require no operands. Atoms can be numbers, letters or strings that fill in templates. Diacritical marks are used to modify atoms. Expressions are built hierarchically by recursively applying templates to existing expressions.

Here are some examples of expressions in Milo:

$$R=\frac{9a_1a_2-27a_3-2a_1^3}{54}$$

$$\frac{\sin x+\cos x}{\tan x}$$

$$B_{\lambda_V}(T)=\frac{6.626\times10^{-27}\,\mathrm{erg}\;2.997\times10^{10}\,\mathrm{s}^{-1}\,\mathrm{cm}^{-3}}{e^{\frac{6.626\times10^{-27}\;2.997\times10^{10}}{1.38\times10^{-16}}}-1}=6.17\times10^{-17}\,\mathrm{erg\,s}^{-1}\,\mathrm{cm}^{-3}$$

Rewriting expressions

In the rewrite stage of the program, Milo can be thought of as a mathematical engine with a WYSIWYG Macintosh interface. You can manipulate and simplify expressions. Any mathematically sensible subexpression can be selected and operated on independently of the rest of the expression. You can define and apply your own rewrite rules that allow complex manipulation. Correctness-preserving transformations can be applied to any part of an expression, often with just a click of the mouse. The rewrite phase of Milo was designed so you could spend more time thinking about the concepts and less time thinking about the details of a problem.

Entering user-defined rewrite rules

A prominent feature of Milo is its ability to perform pattern-matching operations. Using the Rules menu, you can store your own set of rewrite rules and apply them to any expression or subexpression in a Milo window. Sets of rules may be grouped together and stored as menu items in the Rules menu, and cannot be saved when quitting from Milo. Here is an example of a simple rule expression:

$$a+b+a^2+a^3$$

You can create and apply a simple rewrite rule:

$$a=a+1$$

to achieve the following result:

$$a+b+(a+1)^2+(a+1)^3+1$$

The rule may also be written as: $a \leftarrow a+1$, which will be demonstrated later.

Dummy variables, chosen from the Diac menu, are used as wild cards that take on the value of any expression during a substitution. Dummy variables are shown in boldface type in Milo.

Printing

To print a document window, select the print option from the File menu, just as you would expect.

Getting started

If you are an experienced Mac user, you might have the urge to experiment with Milo right away. Now is a good time. If you are feeling less confident, take the time to read the next two chapters. The next chapter shows you how to get Milo running on your Mac, and the following one leads you through five tutorials that demonstrate the steps described above and the power and flexibility of the program.

Chapter 2 Getting Started

What this chapter contains

- Macintosh editing tools
- Milo editing tools
- How to get Milo running on your machine

Macintosh editing tools

Take a moment to make sure you are familiar with editing on the Macintosh. Milo makes use of the basic editing tools built into the Macintosh for both text and symbol editing. The Macintosh editing tools that Milo makes available, including some modifications to the standard Macintosh commands, are listed here as a reference:

Cut, Copy, Paste, Clear, Duplicate and Undo

The following Macintosh commands help manipulate text blocks and equations in Milo and are found in the Edit menu:

Cut Removes a selected block of text or graph and places it in the Clipboard.

Copy Copies a selected block of text or graph to the Clipboard without removing it from the document where it resides.

Paste Inserts a block of text or graph from the Clipboard at a chosen insertion point.

Duplicate Duplicates a selected text or equation block.

Clear Clears the selection range

Undo Cancels the last command typed, and restores the document to its previous state. **The Undo operation works in Equation Block mode only, not in Text block mode**. Though there is no official Redo command, if you select Undo once and then select it again, you will have effectively performed a Redo.

The Scrapbook

The Scrapbook file and the Clipboard file help store and move blocks of text and graphics:

Scrapbook (Apple menu) The Scrapbook is a desk accessory used to store and transport pictures and text.

Menus in general

Menus contain lists of choices. Each pull-down menu on the Mac contains a list of related functions. When a menu item is written in black it is said to be "selectable," meaning that its function can be performed in the particular place you are in the program. If a menu item is "grayed-out," it is not selectable, and inappropriate for the place you are in the program. Menu items that have keyboard equivalents have them listed on the right side of the menu.

Menus in Milo

Milo displays the standard Apple, File and Edit menus, and adds different menus depending on which mode (equation, text or graphics) the program is in. This section outlines the way menus are set up in Milo and describes briefly what they do. Chapters four, five and six provide in-depth information on each of the Milo menu commands.

Text mode

The menus for Text mode:

Style	Alters style and size of font
Font	Changes font families

Equation mode

Commands for entering expressions are contained in the following pull-down menus.

Diac	Diacritical marks like primes, dots, tildes and bars
Templates	The most common objects as well as the Calculus, Trig and More submenus:

Commands for rewriting expressions are contained in the following pull-down menus:

Simp	A range of simplifying techniques
Tfm	A range of correctness preserving transformations
Rules	User-definable rewrite rules to simplify expressions

Using the Option command in the Edit menu, the Calculus, Trig and More submenus may be listed as top-level menus and displayed in Milo's menu bar.

When you insert a template from the Templates menu, the template you have chosen will appear on the screen with **?** prompts in any and all the places that need to be filled in.

Bringing up Milo

If you haven't already done so, you should make a copy of the Milo distribution disk. Put the original Milo disk in a safe (but memorable) place.

Copying the Milo disk

Before you do anything else, lock the Milo distribution disk. This is accomplished by sliding up the black tab on the back of the disk to reveal a hole where the tab was. When the disk is locked, it cannot be erased. There is no reason for you ever to unlock the disk since you only want to use it to make copies.

Copying the disk using a Mac with one floppy disk drive

- Turn on your Macintosh and insert the Milo distribution disk into the floppy drive.
- When you see the disk icon appear on the screen, choose Eject from the File menu and remove the Milo disk.
- Insert a blank formatted 3.5-inch disk into the drive. Refer to your Macintosh manual for instructions on formatting a disk if you are not sure how to do this. Name your blank disk "Milo Work Disk."
- You should now see two icons on your desktop, one called Milo and one called Milo Work Disk.
- Click on the Milo distribution disk, hold down the mouse button and drag its icon on top of the Milo Work Disk icon. When the Milo Work Disk icon turns black, release the mouse button. The Milo Work Disk will automatically eject from the drive and you will be asked to insert the disk called Milo.
- Follow the instructions in the dialog box to swap disks. The more memory (RAM) you have installed in your machine, the fewer disk swaps will be needed to copy the disk.

Copying the disk using a Mac with two floppy disk drives

- Turn on your Macintosh and insert the Milo distribution disk into the internal floppy drive. If you have two internal floppy drives, insert the Milo distribution disk into the top drive.
- When you see the disk icon appear on the screen, choose Eject from the File menu and remove the Milo distribution disk.
- Insert a blank formatted 3.5-inch disk into the second drive, either the external floppy drive, or the bottom internal drive. Refer to your Macintosh manual for instructions on formatting a disk if you are not sure how to do this. Name your blank disk "Milo Work Disk."
- You should now see two icons on your desktop, one called Milo and one called Milo Work Disk.
- Holding down the mouse button, drag the outline of the Milo distribution disk icon on top of the Milo Work Disk icon until the Milo Work Disk icon turns black. Release the mouse button.
- The Milo Work Disk will eject and you will be asked to insert the Milo distribution disk.
- After you insert the Milo distribution disk, a dialog box will appear, asking if you want to completely replace the contents of the Milo Work Disk with the contents of the Milo distribution disk. Click OK.

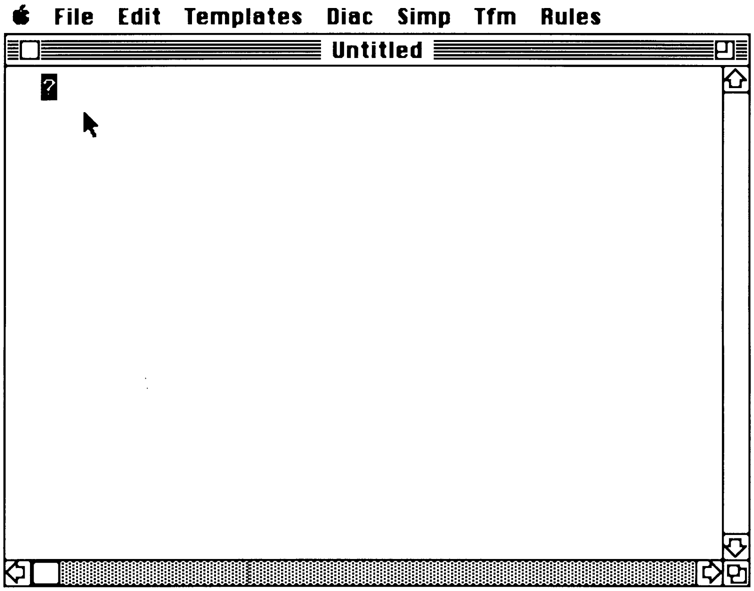
Copying the disk using a Mac with a hard disk

- Boot your Macintosh from the hard disk as you normally would and open your hard disk if it is not already open.
- Create a new folder on your hard disk by selecting New Folder from the File menu. Call the new folder "Milo Folder." Refer to your Macintosh manual for more information on how to do this if you are unsure.
- Insert the Milo distribution disk into the 3.5-inch disk drive and double-click on the Milo distribution disk icon. Select the Milo Application, Tutorials, Backslash.ini and Tables of Formulas icons by dragging a box around them.
- Holding down the mouse button, drag the outline of the selected files on top of the "Milo Folder" icon until the "Milo Folder" icon turns black. Release the mouse button.

Remember to put the original Milo distribution disk in a safe place away from magnets and foul weather, and reboot your Macintosh.

Love at first sight

Open the Milo Folder you created. When first learning Milo, be sure the caps lock key is not engaged. Double-click on the Milo application with the **M** icon inside the folder. You have to wait a few seconds while the program is loaded into memory. The first screen you will see is this:



The Untitled window with a **?** prompt in the upper left corner is where you will build, manipulate and rewrite expressions as well as type text. Graphing expressions takes place in another window. When you are ready to save the "Untitled" window, you can name it just about anything you like.

Fasten your seatbelt, you are now ready to take off. The next chapter contains five tutorials that will help you work through some of the things you can do with Milo.

Chapter 3 Tutorials

What this chapter contains

Step-by-step examples of how to use Milo in the following areas:

- Basic editing
- Differentiating and integrating
- User-defined rewrite rules
- Strings and numbers
- Graphing

This chapter contains six tutorials demonstrating most of the basic features of Milo by presenting some math problems step by step. The Tutorials provide a feel for how to use Milo by example, without explaining the commands in too much detail. The reference chapters four through seven are reserved for a complete explanation of all the Milo commands and their functions.

How to use the tutorials

To get the most from the tutorials provided in this user's guide, you should run the program and go through each step as it is outlined in the tutorials. It is a good idea to duplicate your work in the tutorials at every step. When you have completed a step, select the current expression and choose the Duplicate command from the Edit menu. This will allow you to look back at a trail of your work so you can see where things went wrong or right.

On the Milo distribution disk is a folder containing Guided Tours that correspond to these tutorials. You may, if you so choose, follow along in the guided tours. Just run Milo and open the first guided tour using the Open command in the File menu.

1

Tutorial 1: Editing in Milo

What this tutorial contains

- Entering an expression
- Keyboard equivalents
- Inserting text
- Building Expressions
- Making a copy of the expression
- Selecting the expression
- Manipulating an expression
- Solving the expression for a variable
- Simplifying the expression
- The Solve For command

When you first boot the program, Milo expects the first thing you type to be an expression. The available Milo menus File, Edit, Templates, Diac, Tfm and Rules, reflect the fact that the program is in equation block mode.

Entering an expression

You do not have to type an expression if you don't want, but let's insert the expression 3^2. Pull down the Templates menu and select the "Power" template. It should look like this:

$\boxed{?}^?$

Replace the highlighted **?** with the number 3 by typing the $\boxed{3}$ key on the keyboard to produce this:

$3^?$.

Click on the remaining **?** to highlight it:

$3^{\boxed{?}}$

and type the number $\boxed{2}$ to produce the result:

3^2.

Congratulations! You have just entered an expression in Milo.

To simplify this expression, first select it by pressing the space bar twice. Next, pull down the Simp menu and choose the Simplify command. This will rewrite 3^2 as 9.

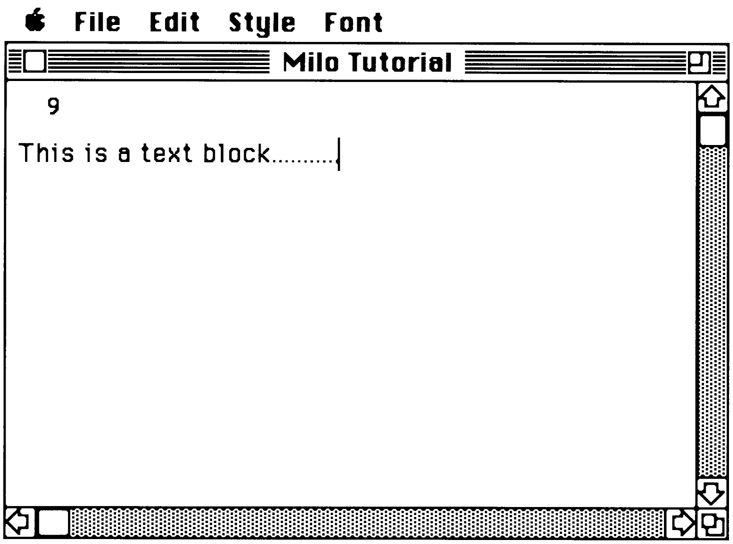
Keyboard equivalents

The sequence described above can be made easier and faster if you use keyboard equivalents for many of the commands. Instead of pulling down the Templates menu and choosing the "Power" command, the same 3^2 expression could have been obtained by using the $\boxed{\wedge}$ key, which is the keyboard equivalent of the Power command. Though this key is actually represented on the keyboard as $\boxed{\text{shift}}$ - $\boxed{6}$, it is represented here as $\boxed{\wedge}$ for clarity. Typing $\boxed{3}\boxed{\wedge}\boxed{2}$ directly at the keyboard produces the same result, since typing the $\boxed{\wedge}$ key is exactly the same as pulling down the Templates menu and selecting the Power command. The keyboard equivalent for the Simplify command is ⌘-A, so instead of pulling down the Simp menu and choosing Simplify, you could just type ⌘-A.

The keyboard equivalents for menu commands are displayed to the right of the command listing in the menu. See the Tips and Techniques chapter for a complete explanation of some keyboard equivalents.

Inputting text

Let's switch gears to get a feel for Milo's text editing capabilities. Pull down the Edit menu and choose the Insert Text Block command. At any time, you can switch between text and equation mode, since the active document window is used to contain both text and expressions. Notice that in text mode, the prompt jumps down a line and changes from a "?" to a blinking "|," and the menu bar changes to display the File, Edit, Style and Font menus. Go ahead and type some text like this:



9

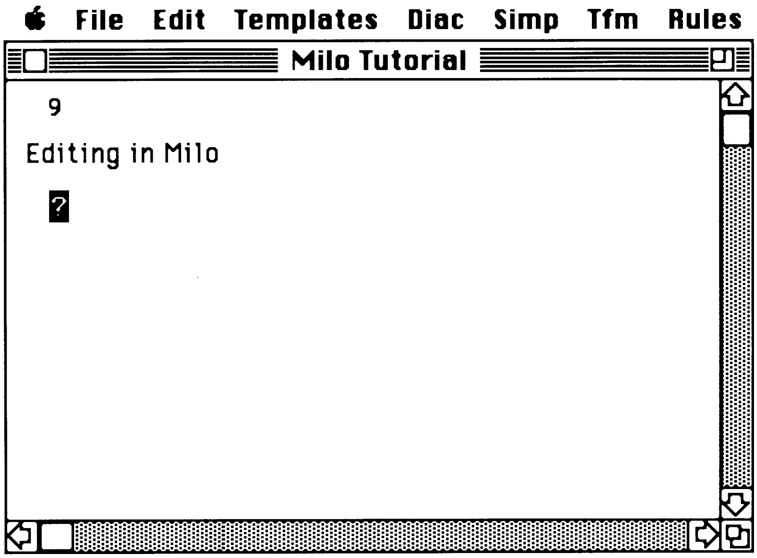
This is a text block..........

In text mode, you can type text and use the menus to change the font or style of what you type. Experiment with typing and selecting text, as well as using the various menu commands. This is exactly the way you use the text mode to enter comments in a Milo document.

You can comfortably type up to 32,767 characters of text per text block, about 4,000 words. In version 1.0 of Milo, the Undo command in the Edit menu is not implemented in text block mode.

Now select everything you just typed in the text block and type in the words: "Editing In Milo." Unlike in equation block mode, **selecting text and typing something new in text block mode deletes everything you had selected and replaces it with what you type**.

Now switch back to equation block mode by pulling down to the Edit menu and selecting the Insert Equation Block command. A faster way to switch from one mode to the other is to use the enter key. Try typing the enter key now. Notice that the prompt again jumps down a line and changes back to a **?**.



9

Editing in Milo

?

Experiment with jumping back and forth from one mode to another with the enter key, but make sure to end up back in Equation mode with the **?** prompt.

Building expressions

Let's take a look at how easy it is to build a more complicated expression, differentiate it, and graph it.

First, make sure Milo is displaying a **?** prompt, and then type:
1 / 3 sp sp X ^ 3 sp sp sp + 1 / 2 sp sp X ^ 2 sp sp sp + 6 X + 8. This will produce the following expression:

$$\frac{1}{3}x^3+\frac{1}{2}x^2+6x+8$$

Making a copy of the expression

Suppose we want to make a copy of the expression we just entered. Try out Milo's Duplicate function by selecting the expression and choosing Duplicate from the Edit menu or by typing ⌘ - D, its keyboard equivalent. You now have a fresh copy of the expression to work with. Make a point of using the duplicate command often to leave a trail of your steps.

Now let's take the derivative of the current equation with respect to "x." With the mouse, select the entire expression by clicking on the left side of the expression and dragging to the right so it looks like this:

$$\frac{1}{3}x^3+\frac{1}{2}x^2+6x+8$$

Now type ⌘ - T to insert the Partial Derivative template to produce this:

$$\frac{\partial}{\partial \boxed{?}}\left(\frac{1}{3}x^3+\frac{1}{2}x^2+6x+8\right)$$

Type an X where you see the **?** prompt to take the derivative with respect to the variable "x" and select the entire expression again. This time, type the Eval Derivative command ⌘ - B and you should see:

$$x^2+x+6$$

Yet Another Expression

Type return and let's learn more about Milo by building another expression:

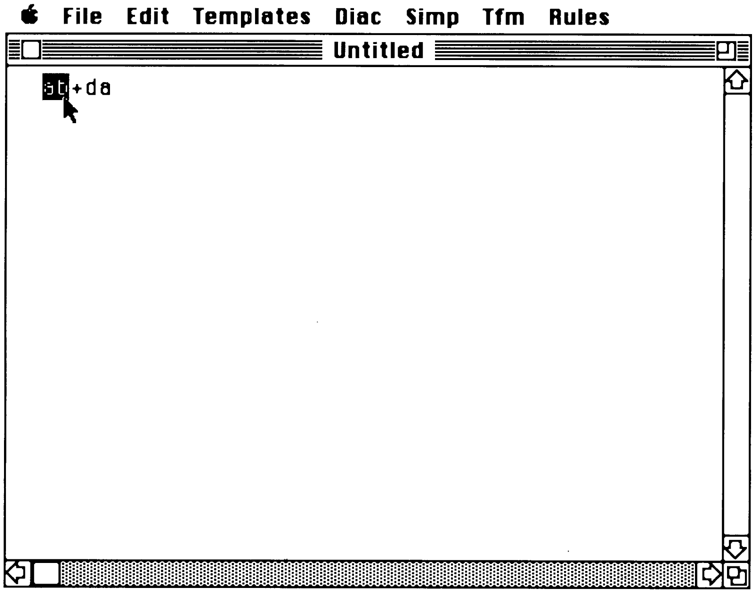
$$\frac{ab+da}{16x}=4x$$

At the **?** prompt in equation mode, type A B + D A, to produce the numerator of the fraction. Now, to divide the numerator by "16x", you have to select what you want the numerator to be and divide it by "16x." If you type the division operator / without selecting the entire numerator, the last atom you typed will be set up as a numerator to a fraction and the expression will look like this:

$$ab+d\frac{a}{?}$$

Selecting the expression

For the entire expression ab+da to be divided by "16x," you have to select the entire expression. Selecting the expression can be done in two ways. First experiment by clicking with the mouse on one side of the expression and dragging toward the other. As you drag, notice the way the selected range jumps to include the next largest subexpression.



ab+da

The alternative to using the mouse is to use the space bar. The space bar is a convenient way of selecting expressions and subexpressions because your fingers never have to leave the keyboard.

To select the expression you just typed, make sure it is not already selected and press the space bar three times. Just like dragging with the mouse, every time you press the space bar, the next largest subexpression is selected:

Pressing the space bar once: ab+da
Pressing it again: ab+da
Pressing it again: ab+da

Milo will never let you select an invalid subexpression like "ab+." You may only select valid subexpressions.

Now type the division operator /. Note that it is applied to the entire selected range, and Milo displays.

$$\frac{ab+da}{?}$$

Type 1 6 X and then = 4 X. **Notice that you do not have to select the entire left-hand side of the expression to have the equals sign apply itself appropriately.**

You should now see an expression that looks like this:

$$\frac{ab+da}{16x}=4x$$

produced from the following sequence of keystrokes: A B + D A sp sp sp / 1 6 X = 4 X

Before moving on, experiment with selecting different parts of the above equation using both the mouse and the space bar until you begin to feel comfortable with the selection process. In the long run, this will save time and frustration. It is important to become proficient with selecting things to use the program efficiently since almost all operations in Milo act on selected ranges.

Manipulating the expression

One of the most useful features of Milo is its ability to manipulate expressions quickly and easily. Let's experiment with this notion. Select the "ab" in the equation you just entered, pull down the Simp menu and select the "Move Term

Right" command. The numerator, which used to read "ab+da," should now read "da+ab."

The Move Right and Move Left commands can be executed from the keyboard using the ⌘ - . combination for Move Right and the ⌘ - , combination for Move Left. The most useful way to move terms around in an equation is with the mouse. Selecting a term in an expression, holding down the ⌘ key and clicking to the left of the term executes a Move Left command, and holding down the ⌘ key and clicking to the right of the selected term executes a Move Right command. This is called "command clicking."

Get a feel for these manipulations. Select the "ab" term in the numerator again, hold down the ⌘ key and click and release the mouse button directly to the right of the equation. You have just executed a Move Right command. The Move Left command is performed in the same way, just select the "ab" again and click to the left to move it back to its original place. Try this operation when just the "b" is selected and watch it change places in the product.

Solving the expression for "a"

Let's solve for "a" in our sample expression. To do this, we want to move the "a" out of the numerator and move the "16x" to the right side of the equation.

Begin by selecting the left-most "a" in the numerator with the mouse. Command-click to the left of the "a." Remember to click directly to the side of the "a," not above or below it. This will factor the "a" out of the numerator and leave the expression:

$$\frac{a\left(b+d\right)}{16x}=4x$$

Now select the "16x" from the denominator, command-click once to the right of it to move it out of the fraction, and again to move it to the right side of the equation. If you stop here, you get this:

$$a\left(b+d\right)=\left(16x\right)4x$$

Simplifying the expression

But why stop now? Select the right side of the equation either by dragging over it with the mouse, or by selecting some part of it and pressing the space bar until the entire right side is selected. Pull down the Simp menu and choose the Simplify command to simplify just the right-hand side to produce:

$$a(b+d)=64x^2$$

Now solve for "a" by selecting the "b+d" subexpression, move it to the right, and simplify the new right-hand side. You should already be familiar with the way most operations (not the equals sign) affect only a selected range, and the possibility of using the ⌘ - A keyboard equivalent for the Simplify command. Your equation should look like this:

$$a=\frac{64x^2}{b+d}$$

Solving the expression for "d"

Let's use the copy you just made to begin solving for "d." First select the denominator "b+d" and move it to the left side of the equation with either two command-clicks or ⌘ - , twice to produce:

$$a(b+d)=64x^2$$

We now want to distribute the "a" into the parenthesis. There are two ways to do this. The first way is to select the whole left side of the equation, pull down the Simp menu and select the Distribute command. The second way is to select just the "a" and move it into the expression to its right by holding down both the ⌘ and the opt keys and clicking inside the parenthesis to the right of the "a." After the distribution has been performed the equation looks like this:

$$ab+ad=64x^2$$

Now move the "ab" term to the right side of the equation by selecting it and command-clicking on its right three times. Notice that it acquires a minus sign in the process:

$$ad=64x^2-ab$$

Isolate the "d" by selecting the "a" still on the left side of the equation and command-clicking it into place on the right. Notice how it is inverted across the equals sign:

$$a=\frac{1}{d}\left(64x^2-ab\right)$$

The Solve For command

An easier way to solve for "d" is to use the Solve For command in the Simp menu. The Solve For command only operates on a selected range, but since there is only one "d" in the above expression, Milo will isolate it. **If you were to select one of the "a's" in the above equation and *Solve For* it, Milo would move the unselected "a" to the other side of the equation along with the other unselected terms.**

Saving, closing and quitting Milo

Before moving on to more advanced features in the next tutorial, it is a good time to learn how to save your work and quit the program. This is done in a standard Macintosh way.

Pull down the File menu and choose the Save command. You will be presented with a dialog box asking for a name for the file, and you can name it "My Tutorial," or virtually anything you like. Click on the save button in the dialog box and then choose Quit from the File menu to return to the Finder.

You will see your new file called My Tutorial in the same folder as the Milo application. If you want to open the file in Milo, just double-click on the My Tutorial icon.

2

Tutorial 2 Differentiating and Integrating

What this tutorial contains

- Differentiating rational functions
- Differentiating trig expressions
- Finding the area bounded by a curve using polynomial integration

This tutorial goes over the steps of how to differentiate expressions in Milo. Polynomial integration is illustrated here. Any integration methods more complicated than polynomial integration are left for the next tutorial that outlines how to use Milo's pattern matcher, and the reference chapters. Some of the examples are graphed to illustrate the how graphs are useful in particular applications.

Before you start this tutorial make sure you are running Milo and have a new "Untitled" window on the screen.

If you are not already running Milo, double-click on its icon in the folder you created. You should see a new window with the **?** prompt highlighted.

If you are running Milo already, type ⌘ - N to create a new "Untitled" window.

Differentiating a rational function

Let's differentiate the following expression:

$$\frac{x^3+2x+1}{x^2+3}$$

First we have to input it into Milo. Start by typing X ^ 3 to get x^3. Press the space bar twice to select what you just typed and enter + 2 X + 1.

Now select everything using either the mouse or the space bar. Type a / to create a fraction, and then type in the denominator: X ^ 2 sp sp + 3. You should be getting a feeling for how Milo operates on a selected range.

Now is another good time to make a copy of the expression using the Duplicate command in the Edit menu. Make sure you first select the expression.

Select the duplicate expression with the mouse and insert the Partial Derivative template by typing ⌘ - T. This also can be done through the Calculus submenu under the Templates menu as well. You should now have an expression that looks like this:

$$\frac{\partial}{\partial \boxed{?}}\left(\frac{x^3+2x+1}{x^2+3}\right)$$

Notice that the insertion point in the derivative template is set to let you specify what to take the derivative with respect to. In this example, type an "x."

Now we want to evaluate the derivative, but first, a useful hint. Select the whole derivative expression, use the Copy command in the Edit menu to copy it to the clipboard, type an = and paste the expression back. You have created an equation and you can now solve the right side while leaving the left side untouched:

$$\frac{\partial}{\partial x}\left(\frac{x^3+2x+1}{x^2+3}\right)=\frac{\partial}{\partial x}\left(\frac{x^3+2x+1}{x^2+3}\right)$$

To evaluate the derivative, simply select everything to the right of the equal sign and type ⌘ - B. This is the keyboard equivalent of the Eval Derivatives command under the Simplify menu. You will see:

$$\frac{\left(3x^2+2\right)\left(x^2+3\right)-2x\left(x^3+2x+1\right)}{\left(x^2+3\right)^2}$$

This is the solution, but if you don't like the way it looks, you can simplify all or part of it using the Multiply Out command in the Simplify menu. Select the numerator and type ⌘ - W (Multiply Out) to produce the better-looking expression:

$$\frac{x^4+7x^2-2x+6}{\left(x^2+3\right)^2}$$

If you feel so inclined, you can simplify the denominator in the same way.

Differentiating a trigonometric expression

Let's differentiate

$$\cos^2(2x)+\sin(3x)$$

Try typing it in on your own. If you get stuck, here is the series of keystrokes that will produce the expression: $\boxed{C}\boxed{O}\boxed{S}\boxed{\text{sp}}\boxed{\wedge}\boxed{2}\boxed{\text{tab}}\boxed{2}\boxed{X}\boxed{\text{sp}}\boxed{\text{sp}}\boxed{\text{sp}}\boxed{\text{sp}}\boxed{\text{sp}}\boxed{+}\boxed{S}\boxed{I}\boxed{N}\boxed{3}\boxed{X}$. Notice how the tab key moves the insertion point to the next prompt. The expression should look like the one above.

Select the expression and insert the Partial Derivative template using either the menu or the $\boxed{\text{⌘}}$ - $\boxed{T}$ keyboard equivalent. Type an $\boxed{X}$ to take the derivative with respect to "x." The expression should now look like this:

$$\frac{\partial}{\partial x}\left(\cos^2(2x)+\sin(3x)\right)$$

To evaluate the derivative, first select the entire expression by pressing the space bar until it is fully selected, then type $\boxed{\text{⌘}}$ - $\boxed{B}$ or pull down the Simplify menu and choose the Eval Derivatives command to produce:

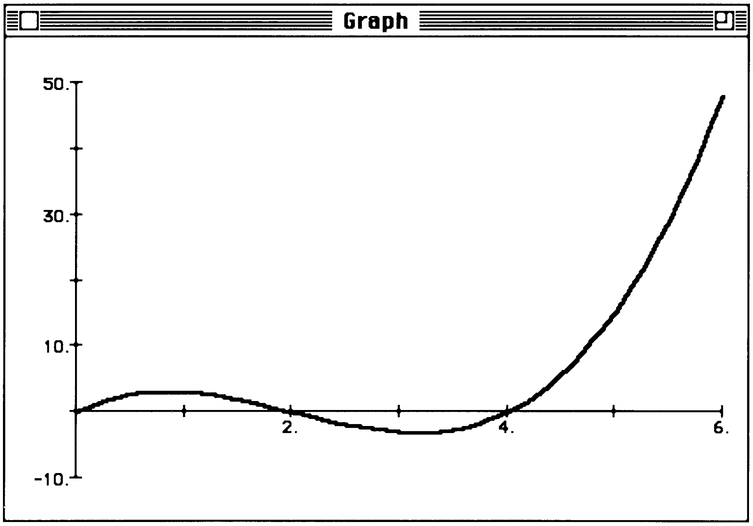
$$-4\sin(2x)\cos(2x)+3\cos(3x)$$

Finding the area bounded by a curve

Let's find the area bounded by the curve: x^3+6x^2+8x and the "x" axis. The way to do this is to graph the expression first, find where the curve crosses the "x" axis, and evaluate a definite integral.

First, enter the expression into Milo: $\boxed{X}\boxed{\wedge}\boxed{3}\boxed{\text{sp}}\boxed{\text{sp}}\boxed{-}\boxed{6}\boxed{X}\boxed{\wedge}\boxed{2}\boxed{\text{sp}}\boxed{\text{sp}}\boxed{\text{sp}}\boxed{\text{sp}}\boxed{\text{sp}}\boxed{+}\boxed{8}\boxed{X}$.

Now graph the expression by selecting it and typing ⌘ - G. In the graphing dialog box, set the domain to go from 0 to 6, then click OK. Do not set any values for the "x" and "y" axes, and leave the number of points set to 50. Here is what the graph should look like:



From the graph, it is easy to tell that the curve crosses the "x" axis at points x=0, x=2 and x=4.

So the area of the portion of the curve lying above the "x" axis is given by:

$$\int_{0}^{2} x^{3}-6x^{2}+8x\,dx$$

and the area of the portion of the curve lying below the "x" axis is

$$\int_{2}^{4} -\left(x^{3}-6x^{2}+8x\right) dx$$

The total area, then, is the sum of these two integrals:

$$\int_{0}^{2} x^{3}-6x^{2}+8x\,dx+\int_{2}^{4} -\left(x^{3}-6x^{2}+8x\right) dx$$

To build this new expression, apply the Definite Integral operator to the existing x^3-6x^2+8x by selecting the expression and typing opt-shift - 1. Use the tab key to cycle through the **?** prompts and fill them in accordingly. With the space bar, select the entire expression, type +, and enter the second integral. Instead of having to type it in again, you can copy the first integral, paste it after the plus sign, and make changes to it. To place parenthesis around an expression, select it, and type a left parentheses. To get the negation operator, type the minus key: -.

When the expression on your screen matches the one above, select it, and choose the Eval Integrals command from the Simp menu (or use ⌘- I) to produce:

$$\int x^3-6x^2+8x\,dx\Big|_{x=0}^{2}+\int -x^3+6x^2-8x\,dx\Big|_{x=2}^{4}$$

Apply the Eval Integrals command four more times until no integrals are left in the expression and it looks like this:

$$\frac{x^4}{4}-2x^3+4x^2\Bigg|_{x=0}^{2}+-\frac{x^4}{4}+2x^3-4x^2\Bigg|_{x=2}^{4}$$

Select everything to the left of the second plus sign:

$$\frac{x^4}{4}-2x^3+4x^2\Bigg|_{x=0}^{2}+-\frac{x^4}{4}+2x^3-4x^2\Bigg|_{x=2}^{4}$$

and type ⌘ - E (Evaluate in the Simp menu). This will produce

$$\left(\frac{x^4}{4}-2x^3+4x^2\Bigg|_{x=2}-\frac{x^4}{4}-2x^3+4x^2\Bigg|_{x=0}\right)+-\frac{x^4}{4}+2x^3-4x^2\Bigg|_{x=2}^{4}$$

If you type ⌘ - / (Eval Substitutions in the Simp menu) you will be left with:

$$(4-0)+\frac{x^4}{4}+2x^3-4x^2\Big|_{x=2}^{4}$$

Repeat the process, this time selecting everything to the right of the first plus sign like this:

$$(4-0)+\frac{x^4}{4}+2x^3-4x^2\Big|_{x=2}^{4}$$

to get:

$$(4-0)+(0--4)$$

Select the above expression and type ⌘ - A (Simplify in the Simp menu) to get 8. So now we know the area between the curve and the "x" axis is 8.

3 Tutorial 3: Using Milo's Pattern Matcher

What this tutorial contains

- Getting the idea
- Using the Rule Tables provided on disk
- Using dummy variables
- Using a rule set
- The Create Menu Item command

Before you start this tutorial, make sure you have a new "Untitled" window on the screen. This can be invoked using the New command in the File menu when running Milo.

Getting the idea

Milo's built-in pattern matcher allows you to rewrite expressions using user-defined rules. Milo does not know how to integrate automatically anything but polynomials. To integrate, for example, a trig function, you have to use Milo's pattern matcher.

This first example is trivial, but it illustrates the basic concept of using the pattern matcher.

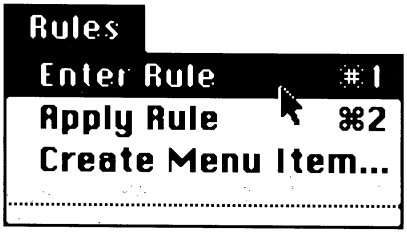
Select the Indefinite Integral template from the Calculus submenu of the Templates menu. Type S I N X and then tab to tab to the next **?** prompt. Type an "x" to integrate with respect to "x."

Try to evaluate $\int sin\ x\, dx$ with the Eval Integrals command in the Simp menu. Remember to select it first, and notice that absolutely nothing happens. But if you know that $\int sin\ x\, dx = -cos\ x$, you can input that as a rule and Milo will be able to perform the integration.

Building a rule is just like building an expression except you use the assignment arrow (←) interchangeably with the equal sign (=). To build a rule that translates $\int sin\ x\, dx$ into $-cos\ x$, select $\int sin\ x\, dx$, type : and then type - C O S X. The colon key shows up on the screen as the assignment operator.

To enter this rule into Milo's pattern matcher, first select the desired rule with the mouse or the space bar, pull down the Rules menu and select the Enter Rule command.

$\int sin\ x\,dx = -cos\ x$



To apply the new rule, select any occurrence of $\int sin\ x\,dx$, pull down the Rules menu and choose the Apply Rule command. Milo will rewrite $\int sin\ x\,dx$ as $-cos\ x$.

This is a trivial example of how to use the pattern matcher and it is admittedly easier in this case to rewrite $\int sin\ x\,dx$ by deleting it and replacing it with $-cos\ x$. But for larger, more complex expressions, the power of the pattern matcher is invaluable.

Using the Rule Tables provided on disk

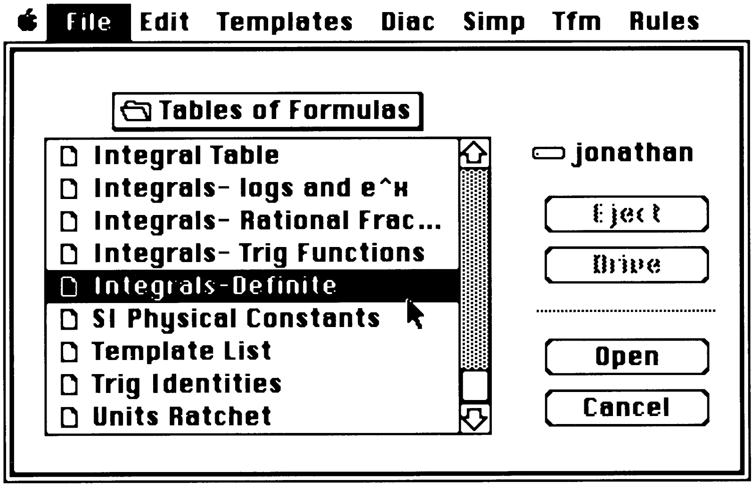
Instead of having to build a rule every time you need one, The Milo distribution disk provides many files of pre-packaged rules. A complete list of the files is included in the reference chapter on rules later in this user's guide. Suppose we create the integral:

$$\int_{0}^{\infty} e^{-\left(\frac{\eta\,\omega}{k\,T}\right)^{2} y^{2}}\,dy$$

Here are the keystrokes used to create the above expression:

opt - shift - 1 E ^ (opt - H opt - W sp sp / K
shift - T sp sp sp sp ^ 2 sp sp ⌘ - − sp Y ^ 2
tab Y tab 0 tab opt - shift - 5

To rewrite this definite integral pull down the File menu, choose the Open command. Select the Milo Folder and you should see a screen that looks like this:



Open the folder called Tables of Formulas. Find the file called Integrals-Definite and open it.

When the file appears on your screen, scroll through it until you find the section called "Some Definite Integrals with the Exponential Function." Select the first rule that will rewrite the integral. It looks like this:

$$\int_{0}^{\infty} e^{-a^2 x^2} dx = \frac{\sqrt{\pi}}{2a}$$

With the above rule selected pull down the Rule menu, choose the Enter Rule command. You may now close the Integral Table file. Note that you do not need to copy the rule into your working Milo document in order to enter it. Now select the expression and type ⌘ - 2 (the keyboard equivalent of the Apply Rule command). See how Milo has rewritten the integral using the rule you entered from the Integral Table file:

$$\frac{\sqrt{\pi}}{2\left(\frac{\omega\eta}{Tk}\right)}$$

The expression

$$\sin(\theta+\theta)$$

can be rewritten using a rule in the file called Trig Identities. Using the Open command in the File menu, open that file and find the rule

$$\sin(\mathbf{A}+\mathbf{B})=\sin\mathbf{A}\cos\mathbf{B}+\sin\mathbf{B}\cos\mathbf{A}$$

Enter this as a rule using the ⌘- 1 keyboard equivalent for the Enter Rule command. Back in the Milo window, select the $\sin(\theta+\theta)$ expression and apply the new rule to get:

$$2\sin\theta\cos\theta\,.$$

Using dummy variables

In the previous example, notice how the variables in the rule are boldface. Boldface variables in Milo are called "dummy variables" and are made to take on the value of any expression. Dummy variables are inserted by typing ⌘ - opt - D, or through the Diac menu.

Here is an example of using dummy variables to recursively subtract the derivative of an expression from the expression. This generates a class of related expressions using a recursion relation. We will start by typing E ^ - T ^ 2 sp sp / 2. This will build the following expression:

$$e^{-\frac{t^2}{2}}$$

Then, using dummy variables to represent the function we want to differentiate, we create the rule to subtract the derivative from the expression times the variable "t:"

$$\mathbf{f} \leftarrow \mathbf{f}\; t - \frac{\partial \mathbf{f}}{\partial t}$$

The rule is created by typing F and making it into a dummy variable by typing ⌘ - opt - D. Next type a colon, : to insert the assignment template and insert another dummy variable **f** to the right of the assignment template in exactly the same way as the first. Then type T as a variable, followed by a minus sign: −. **Do not make "t" a dummy variable.**

Now insert the Partial Derivative template by typing ⌘-T and insert the dummy variable and the variable "t" as follows:

$$f \leftarrow f\ t - \frac{\partial f}{\partial t}$$

All that remains is to enter the rule and apply it. To enter the rule, select it and type ⌘-1. Then select the expression

$$e^{-\frac{t^2}{2}}$$

and type ⌘-2 to apply the rule once. This will produce:

$$t\, e^{-\frac{t^2}{2}} - \frac{\partial}{\partial t} e^{-\frac{t^2}{2}}$$

Before applying the rule again, it will help to simplify the expression a bit. With this new expression still selected, type ⌘-B to evaluate the derivative, and then ⌘-W to multiply out the terms of the expression. This will leave

$$2t\, e^{-\frac{t^2}{2}}$$

You can apply the rule as many times as you like, repeating the three commands in order: ⌘-2, ⌘-B, ⌘-W. When you decide to stop, you may factor the

$$e^{-\frac{t^2}{2}}$$

term out of the expression by selecting it and typing ⌘-F. Use the Simplify command to clean up the result.

Applying the rule six times, factoring and then simplifying produces this expression:

$$e^{-\frac{t^2}{2}}\left(64t^6-480t^4+720t^2-120\right)$$

Using a rule set

In the Tables of Formulas folder on the Milo distribution disk, you will find a file called Units Ratchet. This file contains many rules related to unit conversion. A complete list of all available files in the Tables of Formulas folder is included in the reference chapter on Rules. Open the Units Ratchet file using the Open command in the File menu. The first equation block of the file contains a list of equations that express the relationship between seconds, minutes, hours, days and years. The equation block looks like this:

$$s=3.1688\times10^{-8}\,yr \qquad yr=365.242\,d \qquad d=24\,hr \qquad hr=60\,min \qquad min=60\,s$$

This is called a ratchet because it cycles through the units, representing the same time frame in terms of seconds, minutes, hours, days and years.

Select the entire line just as it is selected above, and enter all the rules at once by typing ⌘ - 1 (Enter Rule in the Rules menu). Now type 2 ' Y R sp sp. That stands for "two years."

Apply the rule set by typing ⌘ - 2 , and then Number Crunch by typing ⌘ - K to produce:

$$730.484\,d$$

Apply the rule again and number crunch:

$$1.75316\times10^{4}\,hr$$

And the same again:

$$1.0519\times10^{6}\,Min$$

And again:

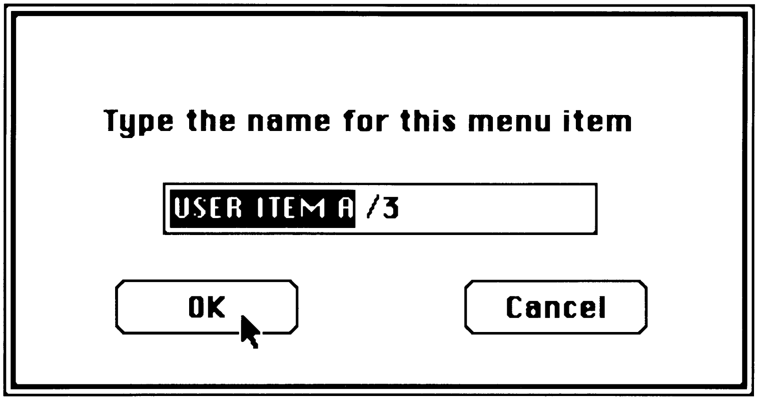
$$6.3114\times10^{7}\,s$$

If you apply the rule set one more time (⌘ - 2), you will cycle back to "2yr."

The Create Menu Item command: using rules as templates

Rules can be created and stored as menu items using the Create Menu Item command in the Rules menu. One use for this feature is to create and store templates.

Build the rule $a \leftarrow a^{2}$, enter it either by using the Enter Rule command in the Rules menu or by typing ⌘ - 1 , and choose the Create Menu Item command from the Rules menu. Remember that the : key produces the assignment operator. The Create Menu Item command brings up this dialog box:



Type the name for this menu item

USER ITEM A /3

Click OK to accept the default name for the rule, or if you like, you may change it to any decent name that fits.

Try applying the rule to this expression: $a^{3}+a^{2}+a+1$. First select the expression, then pull down the Rules menu and choose USER ITEM A. Note the way Milo applies the rule and automatically multiplies the exponents $a^{6}+a^{4}+a^{2}+1$

4

Tutorial 4 Strings & Numbers

What this tutorial contains

- Strings: variables longer than one character
- Scientific notation
- Number crunching

This tutorial demonstrates how to harness more of the power of Milo. Make sure you have the program running on your Mac, with a new "Untitled" window open to begin.

Strings: Variables longer than one character

It is possible in Milo to use variable names that are longer than one character. This is accomplished with the single quote key ' or the backslash key \. If you type a single quote followed by several characters, you define a multiple character variable.

If you are using a program like Smart Quotes that intercepts the single quote key and replaces it with a curly quote, disable it before proceeding.

Now in an equation block type a ' and note that two double quotes: "" appear on the screen with the insertion prompt between the two quotes. Continue by typing K I N E T I C _ E N E R G Y and then press the space bar sp. Note the use of the underscore key to denote a space between words. You have just created a string called "Kinetic_Energy" that will be treated as a single atom.

When you type the first character of the string, a single quote appears just in front of it to let you know it is a string and not an atom. Strings longer than one character are not marked by a single quote, but can be distinguished from several atoms multiplied together because the characters in a string are more tightly packed than individual atoms.

Select the string you just typed and try = 1 / 2 sp sp ' M A S S sp ' Y E L O C I T Y sp ^ 2 You should see:

$$\text{Kinetic\_Energy} = \frac{1}{2}\text{mass velocity}^2$$

With the mouse, select this portion of the expression:

$$\mathrm{Kinetic\_Energy} = \frac{1}{2}\,\mathrm{mass}\ \mathrm{velocity}^2$$

and type an open parenthesis to produce:

$$\mathrm{Kinetic\_Energy} = \frac{1}{2}\left(\mathrm{mass}\ \mathrm{velocity}^2\right)$$

Substitutions

Let's substitute the string "kilogram" for mass and "meters/second" for velocity. Select this part of the equation

$$\mathrm{Kinetic\_Energy} = \frac{1}{2}\left(\mathrm{mass}\ \mathrm{velocity}^2\right)$$

and type the command for the substitution template, opt - \, to produce

$$\mathrm{Kinetic\_Energy} = \frac{1}{2}\left(\mathrm{mass}\ \mathrm{velocity}^2\right)\Big|_{?\,=\,?}$$

Now, remembering to use the single quote key to build a string, type the string "mass" at the first **?** prompt. Use the tab key to jump to the remaining **?** prompt and type the string "kilogram." Select this part of the expression

$$\mathrm{Kinetic\_Energy} = \frac{1}{2}\left(\mathrm{mass}\ \mathrm{velocity}^2\right)\Big|_{\mathrm{mass}\,=\,\mathrm{kilogram}}$$

and use the ⌘ - \ (or choose the Eval substitution command from the Simp menu) to perform the substitution to make the expression look like this:

$$\mathrm{Kinetic\_Energy} = \frac{1}{2}\left(\mathrm{kilogram}\ \mathrm{velocity}^2\right)$$

We can substitute "meters/second" for velocity in the same way. Select everything inside the parenthesis, including the parenthesis themselves. Insert the Substitution template from the Templates menu and fill in the prompts by typing a ' and then the word "velocity" and then another ' followed by M E T E R S sp / ' S E C. Now select the appropriate subexpression so the equation looks like this:

$$\text{Kinetic\_Energy} = \frac{1}{2}\left(\text{kilogram velocity}^2\right)\Bigg|_{\text{velocity} = \frac{\text{meters}}{\text{second}}}$$

And evaluate the substitution to produce:

$$\text{Kinetic\_Energy} = \frac{1}{2}\left(\frac{\text{kilogram meter}^2}{\text{second}^2}\right)$$

Scientific Notation

Numbers in Milo may be displayed in scientific notation. Make sure you are in an equation block and type the following sequence of keystrokes at the **?** prompt: 6 . 0 2 shift - E 2 3. This will produce:

$$6.02 \times 10^{23}$$

Only the "6.02" is part of a string. You can adjust the range of numbers displayed automatically by selecting the Options command in the Edit men (see chapter on Other Features of Milo).

Number Crunching

Enter the expression

$$sin\,\frac{\pi}{3}\;,$$

select it and type ⌘ - K to number crunch it. The atom π may be entered by typing opt - P or \ P I. Any expression that can be evaluated into a number can be operated on by the Number Crunch command.

Enter the keystrokes (X - 1 sp sp sp sp ^ 1 0 to build the expression

$$(x-1)^{10}$$

Note you do not have to type a closing parenthesis since Milo creates both when you type the (key. Select the expression and apply the Partial Derivative template from the Calculus submenu of the Templates menu:

$$\frac{\partial}{\partial x}(x-1)^{10}$$

Now select the entire expression and evaluate the derivative using the Eval Derivatives command in the Simp menu to create the expression:

$$10(x-1)^{9}$$

For an added thrill, select the above expression and use the Multiply Out command in the Simp menu or type its keyboard equivalent ⌘ - W to expand it.

Select this new long expression and apply the Substitution template from the Templates menu. Substitute "e" for "x," and evaluate the substitution as in the example above to produce:

$$10x^{9}-90x^{8}+360x^{7}-840x^{6}+1260x^{5}-1260x^{4}+840x^{3}-360x^{2}+90x-10$$

Finally, select the result and Number Crunch it using ⌘ - K. Since "e" is evaluated as 2.71828 (you can set the number of decimal places using the Options command in the Edit menu), everything in the expression could be converted to floating point numbers and "crunched" to 1305.72.

5

Tutorial 5 Graphing Expressions

What this tutorial contains

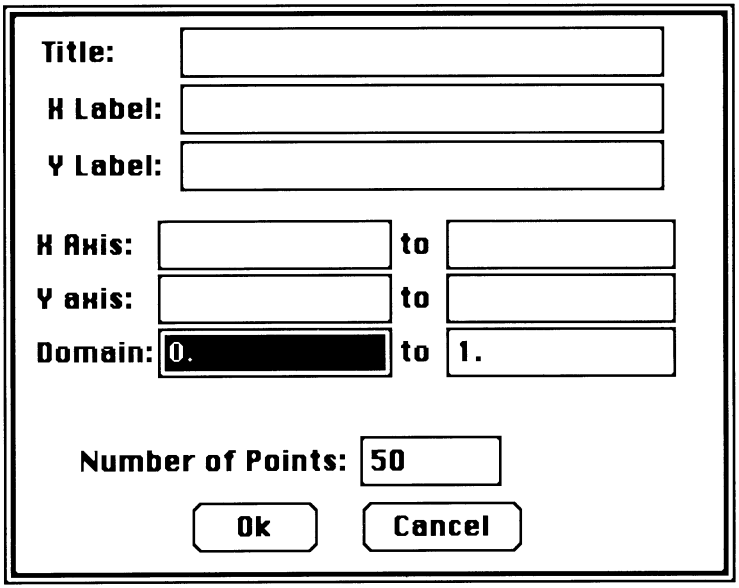
This tutorial demonstrates the graphing features available in Milo. These include:

- Graphing functions of one variable
- Graphing a trigonometric expression
- Zooming in on a graph
- Overlaying graphs
- Graphing a parameterized curve

Before starting, make sure you are running Milo and have a new "Untitled" window on the screen.

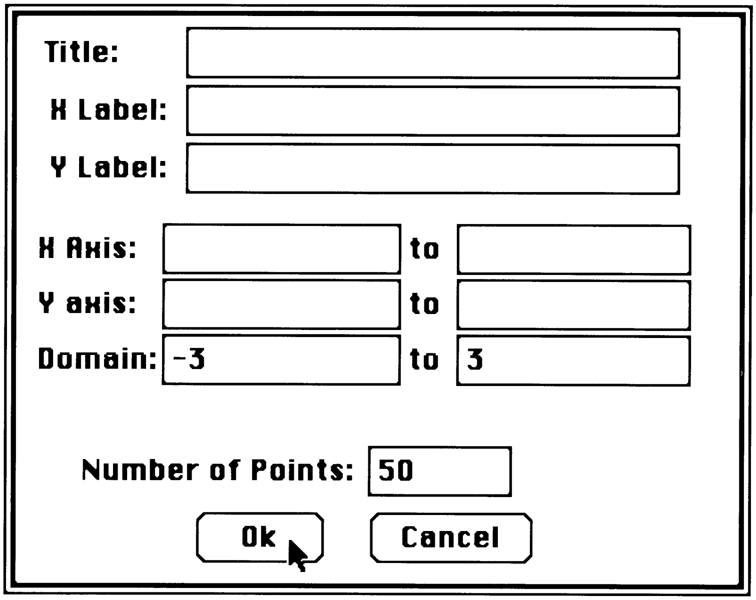
Graphing a function of one variable

To graph the expression $sin\ e^{-x^2}$, first input it into Milo by typing S I N E ^ - X ^ 2, select the expression, and choose the Graph Expression command under the Edit menu. This can also be invoked by typing ⌘ - G. You will be faced with a dialog box that looks like this:



Title:			
X Label:			
Y Label:			
X Axis:		to	
Y axis:		to	
Domain:	0.	to	1.
Number of Points:	50		

Suppose you want to graph $\sin e^{-x^2}$ from -3 to 3. You need not fill in all the values requested by the dialog box because Milo will pick reasonable values for the limits for "x" and "y" and the number of points to plot, if the boxes are left blank. Do supply the domain of the independent variable (in this case "x") and have it run from -3 to 3. Before you click OK, check to make sure the dialog box looks like this:



Title:

X Label:

Y Label:

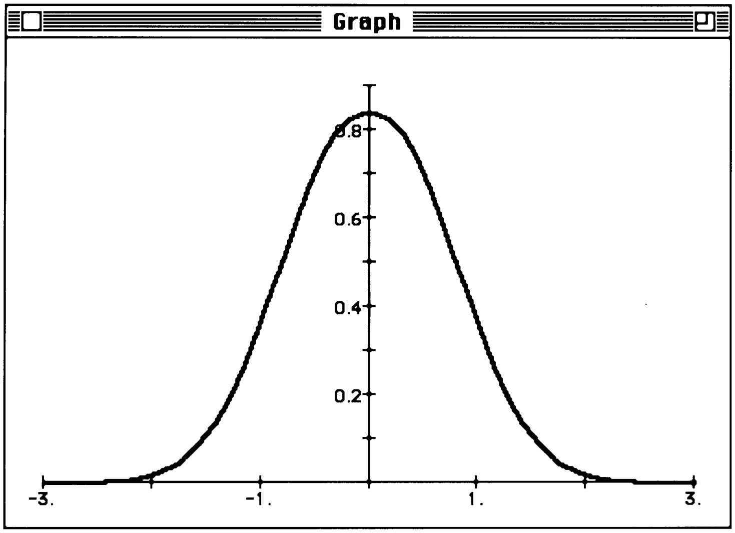
X Axis: to

Y axis: to

Domain: -3 to 3

Number of Points: 50

When you click OK, Milo sets off to graph the 50 points. Each point is displayed as a number in the upper left corner of the screen as it is calculated. This is the way the completed graph should look on the screen:

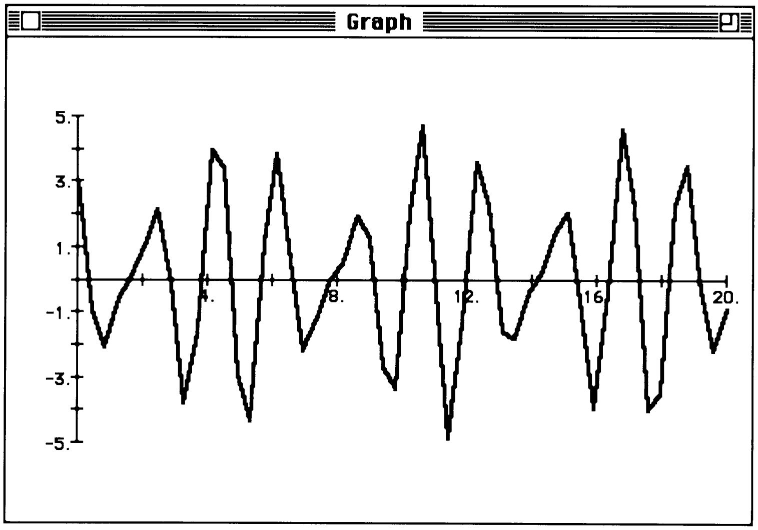


Graphing a trigonometric expression

Using an example from the previous tutorial on differentiating and integrating, let's say we want to graph $-4\sin(2x)\cos(2x)+3\cos(3x)$. This, too, is a graph of a function of one variable. Just select the entire expression with the mouse and type ⌘ - G, or if you are die-hard menu puller, choose the Graph Expression command from the Edit menu, which will bring up the graphing dialog box.

If you have created graphs already during this session, the dialog box will contain the parameters from the last graph Milo drew. Delete them (you can cycle through the boxes with the tab key) and leave the boxes blank. Do not put 0 in any of the boxes, just leave them blank.

In the dialog box, we will set the domain parameter to go from 0 to 20. All the other parameters should be blank. The domain parameter tells Milo what numbers to use as input and is always the same as the value of "x." The parameters shown in the graphing dialog box are explained in the reference chapter on graphing. For now, type a 0 in the first Domain field, then use the tab key to jump to the "To" box in the "Domain" section and set it to 20. Type a return or click OK to produce this graph:



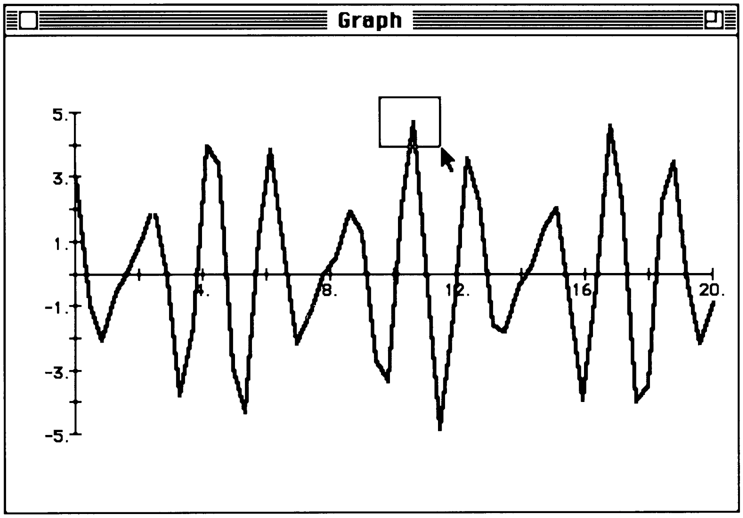
Zooming in on a graph

Let's zoom in on part of this graph to get a closer look at a maximum point. There are two ways to zoom in on a graph.

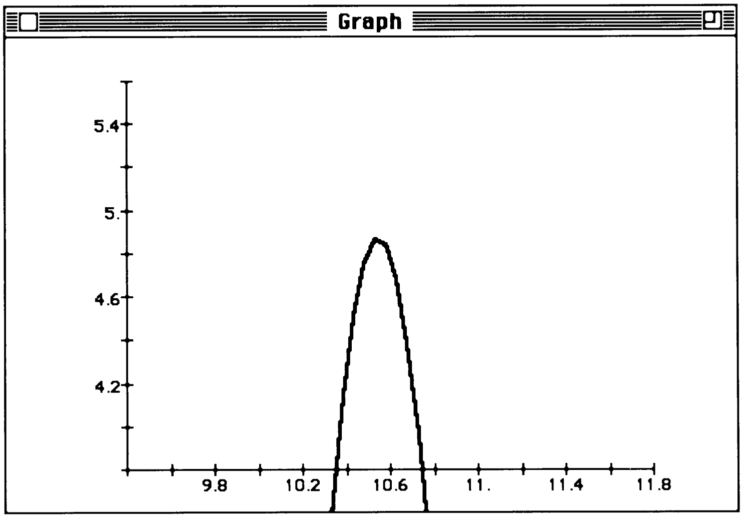
The first way is to once again choose the Graph Expression command from the Edit menu or type ⌘-G and set the "x" and "y" values manually to effectively zoom in on a portion of the graph. This is the long way.

The second and far easier way is with the mouse, to drag a box around the area you want to magnify and let go of the mouse button. Milo will automatically redraw the boxed-in portion of the graph.

With the mouse, select one of the maximum points in the graph above by clicking in the upper left corner and dragging to the lower right:



When you let go of the mouse, the graphing dialog box appears with altered values in most of the parameter boxes. These are the new coordinates for the graph. Click OK and a new graph will be created showing the selected portion of the original graph:



Try zooming in again on the new graph to get an even more accurate measurement. If you like, you can paste this graph into your Milo document using the clipboard. The graph may be resized by clicking in the bottom right corner of the graphing window, though there is no resizing icon present. Once resized, the graph may easily be pasted into a Milo document.

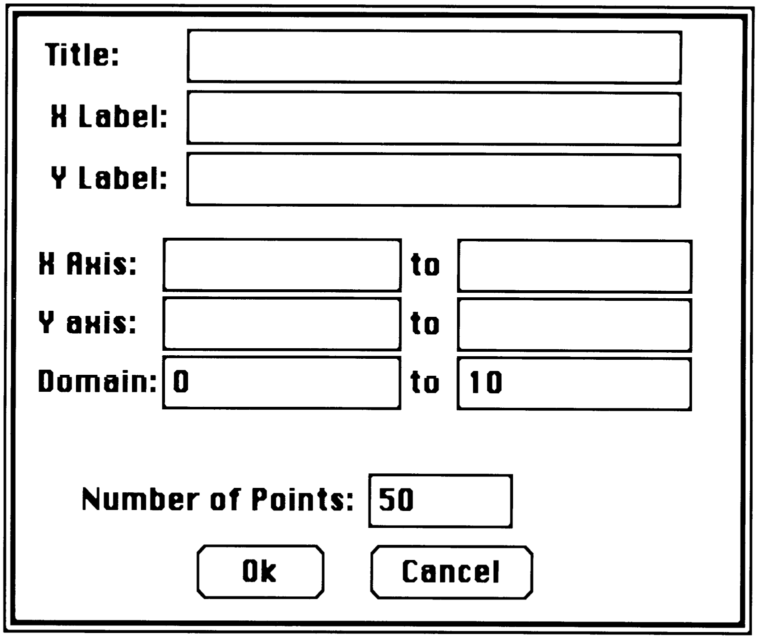
Overlaying graphs

Suppose you want to graph two expressions in the same plot to find out where they intersect. Let's take the trig functions $\frac{\cos x \sin x}{\tan x}$ and $\sin x$.

Build the first expression by typing C O S X sp sp S I N X sp sp sp / T A N X. Now type return and build the second expression by typing S I N X. You should have two expressions on two separate lines.

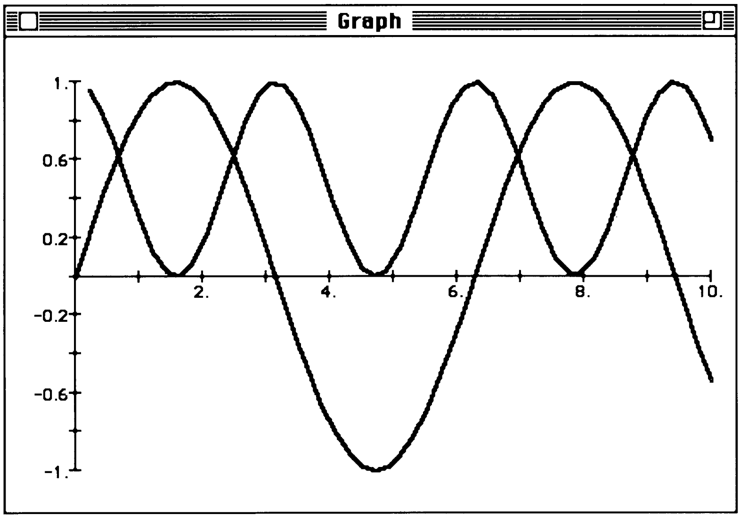
Select the top expression and, with the opt key pressed, drag down to the second expression to select both lines. Note that there is only one variable, "x," in both expressions. Now type ⌘ - G to graph them both at once.

Configure the graphing dialog box as follows, then click OK.



Title:
X Label:
Y Label:
X Axis: to
Y axis: to
Domain: 0 to 10
Number of Points: 50

Milo will respond with the following graph:



You can zoom in on any area of the graph as discussed above, and determine the points of intersection

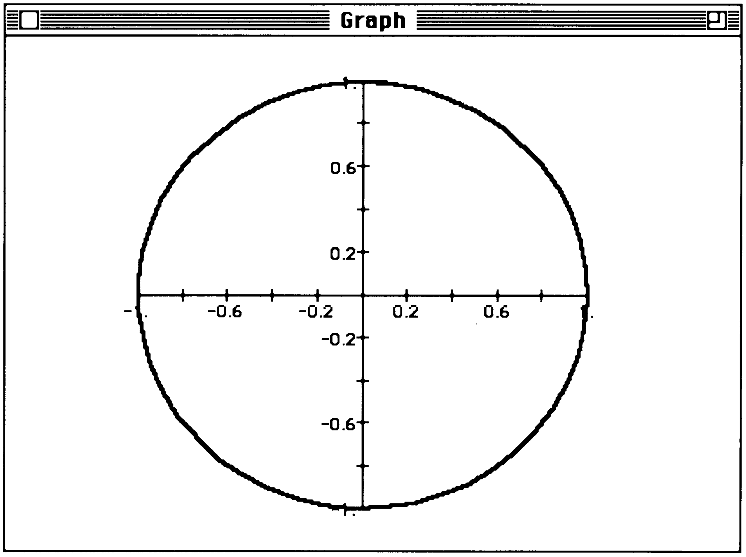
Graphing a parameterized curve

This example shows how to graph a circle and paste it into a Milo document.

Type the expression S I N X. Now type a ;, and then C O S X. The semicolon operator lets you create more than one expression per line. Whenever you try to graph two expressions that are on the same line, Milo assumes you want a parameterized curve, assigning one function to "x" and the other to "y."

With the mouse, select both expressions and type ⌘ - G. Since we are graphing a circle, choose a domain that goes from 0 to 6.5 (approximately 2π), and click OK.

You have produced a circle:



Though there is no visible resizing box at the bottom of this window, you can resize the graphing window by clicking in the lower right corner of the window and dragging it. Shrink the window until it is about two inches square. Type ⌘ - C (or choose the Copy command from the Edit menu) to copy the graph to the clipboard.

Bring the Untitled Milo window to the foreground and paste the graph in at the end (use ⌘ - Y or the Paste command in the Edit menu). It's that simple.

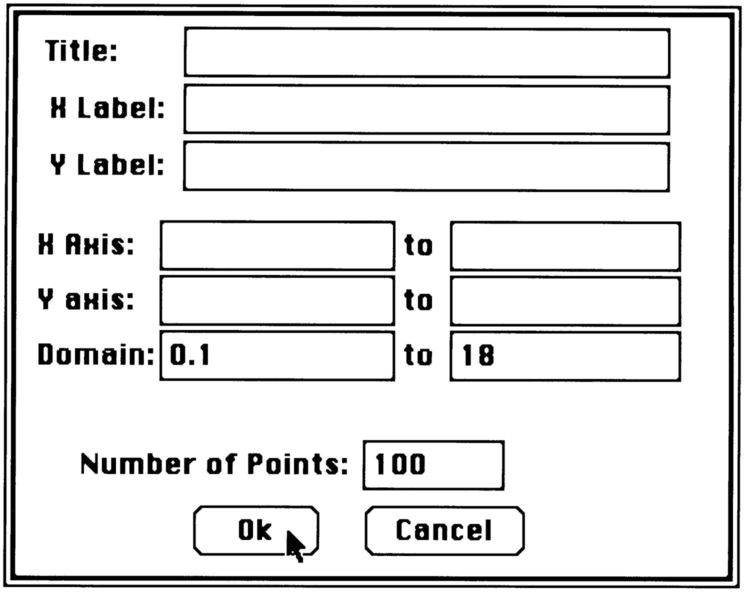
Another parameterized curve

Just for practice and partly for aesthetic reasons, example of a graph.

Enter the following two expressions and separate them in the same equation block by typing a semicolon.

log t *sin* t   *log* t *cos* t

Select them both by clicking and dragging and use ⌘ - G to bring up the graphing dialog box. Set up the dialog box so it looks like this:



Title:

X Label:

Y Label:

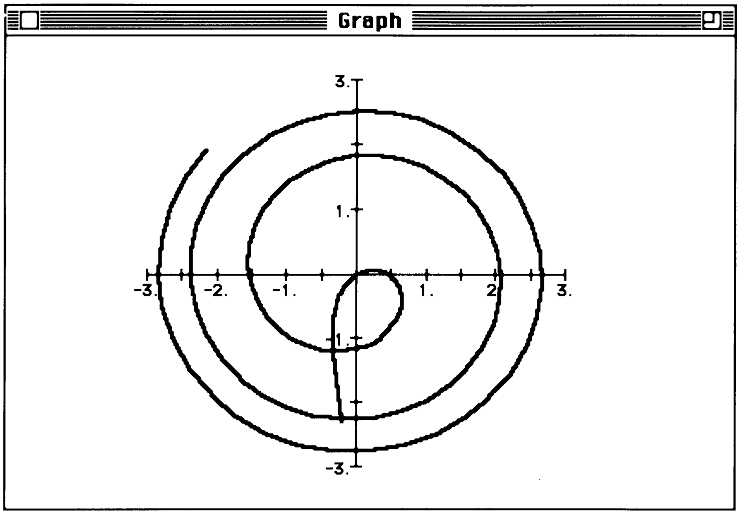
X Axis: to

Y axis: to

Domain: 0.1 to 18

Number of Points: 100

You may have to delete some fields and leave them blank. Notice that the domain parameter is set to go between 0.1 and 18.



Chapter 4 Input Reference

What this chapter contains

Inputting expressions in Milo

- Atoms
- Polynomials
- Unary operators
- N-ary operators
- Vectors, matrices, superscripts and subscripts
- Calculus templates

This chapter describes in detail every Milo command relating to the input of mathematical information. Each entry in the chapter describes a Milo command, provides standard macro syntax and information about how to use the command.

The format of this chapter

Each input command in this chapter is listed as it appears in Milo's menus. Relevant information such as keyboard equivalents and macros, are listed directly below the command name. The location of the command in the menus is shown using the greater-than symbol > to denote submenus and finally to point to the command name. This is followed by a complete description of the command.

Milo's ability to graph expressions is not discussed in this chapter, but reserved for the reference chapter on graphing.

Atoms

The simplest objects from which Milo expression are built up are called atoms. They are objects without any operands.

The prompt

The first atom you encounter in Milo is called a prompt and is displayed as a bold-faced question mark. The following line is a prompt.

?

The prompt is used as a place holder to let you know that Milo expects an expression there. For example when the Summation template is chosen from the Templates menu, the following template is inserted:

$$\sum_{?=?}^{?} ?$$

The tab key cycles the selection range through the prompts in the template so they may be filled in at leisure.

Character atoms

Typing any alphabet character over a prompt replaces the prompt with that character. Upper and lower case characters represent different atoms, and certain combinations of the opt and shift keys produce Greek character atoms as shown in the table below.

Greek Name	Lower case	Upper case	Script
Alpha	α opt - A	A shift - A	
Beta	β opt - B	B shift - B	
Gamma	γ opt - G	Γ opt - shift - G	
Delta	δ opt - D	Δ opt - shift - D	
Epsilon	ε opt - shift - E	E shift - E	
Zeta	ζ opt - Z	Z shift - Z	
Eta	η opt - H	H shift - H	
Theta	θ opt - Q	Θ opt- shift - Q	ϑ opt shift ;
Iota	ι opt - shift - I	I shift - I	
Kappa	κ opt - K	K shift - K	
Lambda	λ opt - L	Λ opt- shift - L	
Mu	μ opt - shift - M	M shift - M	
Nu	ν opt - shift - N	N shift - N	
Xi	ξ opt - X	Ξ opt- shift - X	
Omicron	o O	O shift - O	
Pi	π opt - P	Π opt- shift - R	
Rho	ρ opt - R	P shift - P	

Sigma	σ	opt - S	Σ	opt - shift - S	
Tau	τ	opt - T	T	shift - T	
Upsilon	υ	opt - U	Y	shift - U	
Phi	ϕ	opt - F	Φ	opt - shift - F	φ opt shift U
Chi	χ	opt - C	X	shift - C	
Psi	ψ	opt - Y	Ψ	opt - shift - Y	
Omega	ω	opt - W	Ω	opt - shift - W	

The Infinity symbol, also an atom, can be invoked by typing opt - shift - 5 or by using the standard macro \inf. The Key Map command in the Edit menu provides a list of available atoms from the keyboard.

Atoms with special meaning in Milo

The following atoms are evaluated by Milo
i is the complex number ι
e is evaluated as 2.71826
π is evaluated as 3.14159

The precision of the decimal place is determined by the Option command in the Edit menu. (See the chapter on Other Features of Milo).

Integers

Keyboard: Keyboard digits
Menu: None
Maximum Integer Length: $2^{31}-1$

Typing a digit at the **?** prompt replaces the prompt with that integer.

Integers are represented as signed 32-bit values and no overflow checking is performed during either simplification or typing. This will produce incorrect results when dealing with large integers. For example, simplifying 2^{32} in Milo yields 0. Typing 9 8 7 6 5 4 3 2 1 0 produces1286608618. Milo behaves as expected until you type the final digit, 0, which overflows the integer. To work happily with numbers greater that 2^{31}, use floating point arithmetic, described in the following section on strings.

Strings

Keyboard: ' (single quote)
Menu: Templates > More > String
Maximum String Length: 61 characters

Typing the three atoms "a b c" at the **?** prompt yields an expression that Milo interprets as the product of three character atoms. For a character string to be represented as an atom a single quote is used as a prefix. The insertion point then becomes a vertical bar between two double quotes and any typing you do will be input into a string that can have at most 61 characters. Thus, the string "abc" is obtained by typing ' A B C

Strings are distinguished from products of atoms by their character spacing. Strings are condensed with hardly any space between characters. Products have at least two empty pixels between each character:

String: abcdef
Product: a b c d e f

The string containing the single letter "A" is different from the character atom "A." To illustrate this difference, strings of a single letter are displayed with the single quote visible like this: 'A.

Besides allowing you to create multi-character variables, there are two special uses of strings. One is to display floating point numbers and the other is to implement a limited macro facility available in the Backslash.ini file on the Milo distribution disk.

Floating point numbers

Keyboard: . period (see strings)

Menu: None

Internal Precision: 18 digits

Floating point numbers are displayed as strings. Typing a decimal point at the **?** prompt produces the string "0." followed by an insertion point. Similarly, typing a decimal point after an integer converts the integer into a string terminated by a decimal point. So to enter the number 1.2, you type 1 . 2.

Scientific Notation

Keyboard: shift-E
Menu: None

Typing shift - E after an integer or a floating point number applies a "times ten to the" operator after the number, and leaves the insertion point in the exponent.

So typing 1 shift-E produces 1 x $10^?$, where the 1 is an integer. Typing 1 . 2 shift - E 8 produces 1.2 x 10^8 where the "1.2" is a string and the 8 is an integer.

Standard Macros

Keyboard: \
Menu: None
Maximum Length: 61 characters

Typing the backslash key, \ , which is equivalent to typing ' \, produces a special string beginning with the backslash character. Typing a space after a string beginning with a backslash causes Milo to do a table lookup on the entries loaded from the file "Backslash.ini" on startup. Milo will replace the string, if it is listed in the Backslash.ini table, with a macro expansion from that table (see Tips and Techniques). **If the caps lock key was held down when Milo was booted, this feature has been disabled.**

For example, typing \ P I sp produces the atom π after the sp is typed.

Polynomials

Polynomials are constructed from the operations of addition, negation, multiplication, division and exponentiation. The commands in the simplify menu treat the polynomial templates using standard algebra.

Addition

Keyboard: +
Menu: None

At the prompt, typing A + yields: a + ?. After you type the +, Milo expects another summand and displays a prompt to ask "plus what?" Plus is an n-ary operator since there may be an arbitrary number of terms in the sum:

$$-a+\frac{c}{d}+b-x^2$$

Addition in Milo operates on any selected range.

Negation and subtraction

Keyboard: − , opt - −
Menu: Templates > Negation

At the prompt, typing − yields: -?. This is unary negation. Typing A − at the prompt yields a - ?. This looks like a binary subtraction, but is stored internally as a + -?. This is important to remember since you can select the -? as single object, but you can never select just the +?.

When the selection is a blackened range, typing − is interpreted by Milo as subtraction. To negate a selected range, type opt - − or select the Negation template from the Templates menu. Milo has no binary subtraction, only a unary negation. This is usually transparent since typing A − B looks like you would expect it to, but internally it is stored as a sum of two objects: the character "a" and a unary negation of the character "b."

Division

Keyboard: /
Menu: None

At the prompt, typing A / yields $\frac{a}{?}$. The division operator operates on any selected range.

Exponentiation

Keyboard: ^
Menu: Templates > Power

The exponentiation operator is entered from either the Templates menu or as the ^ key from the keyboard. At the **?** prompt, typing A ^ yields $a^?$ The exponentiation operator operates on any selected range.

Multiplication

Keyboard: *
Menu: None

At the prompt, typing A * B yields "ab." The * operator is not necessary, however, since multiplication is the assumed operation when typing adjacent atoms, so that typing A B also yields "ab."

When the insertion point is a blackened selection range, typing does not replace the selection range, it acts on it. So that by typing A sp B, the "b" does not replace the "a," but instead is multiplied by it. To replace an expression requires an extra keystroke (delete or backspace) and multiplying two subexpressions does not require typing the * character.

For example, typing − 2 X ^ 2 sp sp sp sp + A / B sp sp X + 1 2 produces the following polynomial:

$$-2x^2+\frac{a}{b}x+12$$

Unary Operators

Unary operators are those templates which take exactly one subexpression as an operand. Atoms have no arguments. Addition and multiplication are n-ary since they take 2 or more arguments. Division and exponentiation are binary, while negation is unary. Other unary operators are:

- √x the **square root** operator,
- n! the **factorial** operator
- |x| the **absolute value** operator
- (x) the **identity operator,**
- <x> the **expectation value** operator
- $\overline{x}$ the **overbar** operator
- log x the **natural logarithm** function
- sgn x the **signum** function
- sin x and all the other trig and inverse and hyperbolic trig functions.

Negation -?

Keyboard: ⌘ - −
Menu: Templates > Negation
Rewrite Commands: None
Standard Macro: None

The Negation operator is entered either from the Templates menu or by typing the ⌘ - − key combination. The Negation template operates on a selected expression or an unselected expression that immediately follows the insertion point. After inserting the Negation template, Milo leaves the insertion point before the expression so typing the backspace key will remove the minus sign. This template is different from typing just the minus key, −, which is used for subtraction, a short way of typing + −. **The Negation template is not used for subtraction.**

Square Root √?

Keyboard: opt - 5
Menu: Templates > Square Root
Rewrite Commands: Simplify, Number Crunch
Standard Macro: \sqrt

The Square Root operator is entered either from the Templates menu or from the keyboard with the opt - 5 key combination. It operates on a selected expression or an unselected expression that immediately follows the insertion point.

After inserting the template, Milo leaves the insertion point before the expression so typing the backspace key will remove the radical. When the insertion point directly follows an expression, typing opt - 5 multiplies the previous expression by a new radical and leaves the insertion point inside the radical with the selected range over the new prompt.

Milo treats the Square Root operator as the principal 1/2 power. This may lead to incorrect results when simplifying roots. For example

$$\sqrt{x^2}$$

simplifies to "x" when it should simplify to the absolute value of "x," or "|x|."

Factorial ?!

Keyboard: !
Menu: Templates > Factorial
Maximum Size: 12!, or use floating point.
Rewrite Commands: Evaluate, Number Crunch
Standard Macro: \fact

The Factorial operator is entered either from the Templates menu or by typing the exclamation mark ! from the keyboard. It operates on a selected expression or can be inserted directly at a **?** prompt. After inserting the Factorial template, Milo leaves the insertion point before the expression so typing the backspace key will remove the factorial.

When the insertion point is before an expression, typing ! will create a new factorial that is multiplied by the expression that follows it. The selected range is always left over the new **?** prompt.

The Factorial operator will insert parenthesis around a selected range when necessary.

Milo treats the Factorial operator in the usual way. 0!=1!=1 2!=2 3!=6 4!=24 For 5! through 12!, the Evaluate command under the Simp menu will expand the factorial.

Any factorial larger than 12 overflows the 32 bit integers, so the Evaluate command will not rewrite them, but the Number Crunch command under the Simp menu will use Sterling's approximation to approximate the factorial as a floating point number. (See the reference chapter on manipulating expressions).

Absolute Value |?|

Keyboard: |
Menu: Templates > Absolute Value
Rewrite Commands: Evaluate, Number Crunch
Standard Macro: \abs

The Absolute Value operator is entered either from the Templates menu or by typing the | character from the keyboard. It operates on a selected expression, or can be inserted directly at a **?** prompt. After inserting the absolute value template, Milo leaves the insertion point inside the absolute value so typing the backspace key will remove the template. When the insertion point is before or after an expression, a new absolute value template is inserted and multiplied to any adjacent expression. The selected range is left over the new prompt.

The rewrite commands that recognize this template are Number Crunch and Evaluate. Number Crunch takes the absolute value when the argument is a number. Evaluate takes the determinate of a matrix when its argument is a 2x2 matrix. Determinates of 3x3`or larger matrices are not programmed directly into Milo's code but may be calculated using Milo's pattern matcher.

Identity (?)

Keyboard: [or (
Menu: Templates > Identity
Rewrite Commands: Evaluate, Number Crunch
Standard Macro: \id

The Identity operator is entered either from the Templates menu or by typing the [or (characters from the keyboard. It operates on a selected expression or can be inserted directly at a **?** prompt. After inserting the identity template, Milo leaves the insertion point inside the parenthesis so typing the backspace key will remove the template. When the insertion point is before or after an expression, and a new identity template is inserted it is multiplied to any adjacent expression. The selected range is left over the new prompt.

The Identity template is used by other templates when necessary. It has no mathematical content and is used simply to clarify what may otherwise be an ambiguous screen display.

Overbar $\overline{\boxed{?}}$

Keyboard: None
Menu: Templates > Overbar
Rewrite Commands: None
Standard Macro: \overbar

The Overbar operator is entered from the Templates menu and has no keystroke associated with it by default, though a keyboard equivalent can be assigned to it using the Command Key command under the Edit menu (see Tips and Techniques).

The Overbar template is different from the diacritical mark which puts a bar over an atom (see Diacritical Markings, this chapter). The diacritical mark is an attribute of an atom, while an Overbar is a template that can be applied to any expression. Standard uses of this symbol include the complex conjugate, logical negation and mean value, depending on the context. There are no built-in rewrite rules in Milo that apply to this template.

Expectation $\langle \boxed{?} \rangle$

Keyboard: None
Menu: Templates > Expectation
Rewrite Commands: None
Standard Macro: \expect

The Expectation operator is entered from the Templates menu and has no keystroke associated with it by default though one can be assigned to it using the Command Key item under the Edit menu.

There are no built-in Milo rewrite rules which apply to this template.

Trigonometric functions

Keyboard: None
Menu: Templates > Trig > sin, cos, sin etc...
Rewrite Commands: None
Standard Macro: None

A submenu of the Templates menu, the Trig menu contains all the trig, arc-trig, hyperbolic trig, and arc-hyperbolic trig functions along with the natural log, signum function and sgn, because they are treated by Milo similarly to the trigonometric functions. They can be entered through the Trig submenu or by typing the characters that make up their names. The natural log operator is called "log" in Milo since there is no built-in log base 10.

After typing [S] [I] Milo interprets the expression as a product of two atoms. When an [N] is typed after that, Milo parses the three characters as *sin*, drawn in italics to show that they constitute a single entity. The **?** prompt following the trig template is left selected. Typing an [H] here will turn *sin* into *sinh* and *cos* into *cosh* etc.

The trigonometric functions are treated in the standard manner. Milo knows that log e=1 log 1= 0; but doesn't know anything about trigonometry. Under the Tfm menu, the Trig -> Base Trig will write *tan*, *csc*, *sec*, *cot* in terms of *sin* and *cos*. The Trig-> Exponential command will rewrite trig expressions in terms of sums of exponentials. Other trigonometric identities are available as rules in the Tables of Formulas folder on the Milo distribution disk that may be applied using the pattern matcher.

N-ary Operators

Substitution ?|? = ?

Keyboard: [opt] - [\]
Menu: Templates > Substitution
Rewrite Commands: Evaluate, Eval Substitutions
Standard Macro: None

When an expression is selected or the insertion point directly follows an unselected expression, the Substitution template can be applied to that expression using [opt] - [\]. If the insertion point is directly before an expression and the substitution template is inserted, Milo will put that expression in the substitution field.

The Evaluate and Eval Substitutions commands in the Simp menu treat the Substitution operator as follows: In $a\big|_{b=c}$, all occurrences of "b" in "a" are lexically replaced by "c."

Eval Between $\boxed{?}\Big|_{?=?}^{?}$

Keyboard: None
Menu: None
Rewrite Commands: Evaluate
Standard Macro: \between

The Eval Between template is set to perform two substitutions on an expression and the Evaluate command in the Simp menu subtracts the second result from the first.

The Evaluate command treats the between operator as follows:

$$x\Big|_{expr\ =\ begin}^{end} \leftarrow x\Big|_{expr\ =\ end} - x\Big|_{expr\ =\ begin}$$

Summation $\sum_{?=?}^{?}\boxed{?}$

Keyboard: opt - shift - S
Menu: Templates > Summation
Rewrite Commands: Evaluate, Eval Sums or Products
Standard Macro: \sum

The Summation template can be inserted through the Templates menu or by using the opt - shift - S keyboard combination.

The simplify commands Evaluate and Evaluate Sums or Products treat the summation operator as follows. In the expression

$$\sum_{b=c}^{d} a$$

all occurrences of "b" in "a" are lexically replaced by "c"and the new expression is added to the existing one. Then "c" is incremented and the process is repeated.

Holding down the shift key while selecting the Eval Sums or Products command will expand the sum or product provided both the limits are integers and their difference is less than 101.

Product $\prod_{?=?}^{?} ?$

Keyboard: opt - shift - P
Menu: Templates > Product
Rewrite Commands: Evaluate, Eval Sums or Products
Standard Macro: \prod

The simplify commands Evaluate and Evaluate Sums or Products treat the Product operator as follows. In the expression below all occurrences of "b" in a are lexically replaced by "c" and the new expression is multiplied to the existing one. Then c is incremented and the process is repeated.

$$\prod_{b=c}^{d} a$$

Holding down the shift key while selecting Eval Sums will completely expand the product provided that the limits of the product are both integers and their difference is less than 101.

Limit $\lim_{? \to ?} \boxed{?}$

Keyboard: None
Menu: None
Rewrite Commands: None
Standard Macro: \lim

The Limit operator is purely syntactic, no rewrite commands currently attach any meaning to it.

Log Base b $log_{?} \boxed{?}$

Keyboard: None
Menu: None
Rewrite Commands: Evaluate
Standard Macro: \logb

The Evaluate command $log_{b} x$ rewrites as follows:

$$log_b x \leftarrow \frac{log\ x}{log\ b}$$

The Number Crunch command in the Simp menu evaluates the Log Base b template numerically if its arguments are numbers.

Vectors, matrices, super/subscripts

Div (Divergence) $\nabla \bullet \boxed{?}$

Keyboard: opt - shift - 6 opt - ;
Menu: Templates > More > Div
Rewrite Commands: None
Standard Macro: \div

Grad (Gradient) $\nabla \boxed{?}$

Keyboard: opt - shift - 6
Menu: Templates > More > Grad
Rewrite Commands: None
Standard Macro: \grad

Curl $\nabla \times \boxed{?}$

Keyboard: opt - shift - 6 opt - shift - z
Menu: Templates > More > Curl
Rewrite Commands: None
Standard Macro: \curl

Laplacian $\nabla^2 \boxed{?}$

Keyboard: None
Menu: Templates > More > Laplacian
Rewrite Commands: None
Standard Macro: \lap

The Div, Grad, Curl, and Laplacian operators are the standard vector calculus derivatives. The Rectangular, Cylindrical, and Spherical commands listed in the Tfm menu, expand Div, Grad, and Curl in each coordinate system. Div and Curl will expand only when their argument is a three-element row or column matrix. The Laplacian template has no built-in meaning to any of Milo's rewrite commands.

$$\nabla f \qquad \nabla \bullet \vec{E} \qquad \nabla \times \vec{B} \qquad \nabla^2 \Phi$$

Dot product ?•?

Keyboard: opt - ;
Menu: Templates > More > Dot
Rewrite Commands: Matrix Algebra
Standard Macro: \dot

See Cross product below.

Cross product ?×?

Keyboard: opt - shift - Z
Menu: Templates > More > Cross
Rewrite Commands: Matrix Algebra
Standard Macro: \cross

The Dot and Cross operators are the standard vector products. The Matrix Algebra command under in the Tfm menu evaluates the Dot or Cross product when both of its operands are three-element row or column matrices. No other items affect them.

$\vec{A}\bullet\vec{B} \qquad \vec{v}\times\vec{B}$

Matrix [?]

Keyboard: opt - M
Menu: Templates > More > Matrix
Rewrite Commands: Matrix Algebra
Standard Macro: \matrix

Applying the Matrix template creates a 1x1 matrix. Typing return in a matrix creates a new row. Typing , in a matrix moves to or creates a new column.

The Matrix Algebra command in the Tfm menu will multiply or add matrices of the appropriate rank and multiply terms into a matrix.

Upper Index ? ?

Keyboard: ⌘ - H
Menu: Templates > More > Upper Index
Rewrite Commands: None
Standard Macro: None

Lower Index ? ?

Keyboard: ⌘ - L
Rewrite Commands: None
Standard Macro: None

Menu: Templates > More > Lower Index

The Upper and Lower Index templates, when applied to expressions which do not already have indices, will add an upper or a lower index to the expression. The Upper Index template is different from an exponent, though it may look the same. The index templates are n-ary operators that themselves can take on an arbitrarily number of upper and lower indices.

If the insertion point is after an index, typing , will add another index. If the insertion point is after an index and either command is invoked it will add an index to the index. If the insertion point is after an upper index and you want to add a lower index to the whole index expression then select the expression and type ⌘ - L to add a lower index. This allows for fast typing of complicated expressions.

For example, typing the following five sequences of keystrokes

1) opt - shift - G ⌘ - H opt - A , opt - B
2) X ⌘ - H J
3) shift - N ⌘ - L X ⌘ - L I
4) shift - R ⌘ - H opt - A , opt - B sp sp ⌘ - L opt - G
5) opt - shift - G ⌘ - H opt - A ⌘ - H ' + sp sp ,
opt - B ⌘ - H ' −

produces the following results:

1) $\Gamma^{\alpha\beta}$ 2) x^{j} 3) $N_{x_{i}}$ 4) $R_{\gamma}^{\alpha\beta}$ 5) $\Gamma^{\alpha^{+}\beta^{-}}$

Calculus Templates

Milo has templates for four kinds of derivatives, three kinds of differentials and two kinds of differential operators.

Everything but the four derivative templates is used for notation purposes only. None of the other derivative templates can be evaluated in Milo unless you create a specific rule to do so with the pattern matcher.

Total derivative $\frac{d?}{d\boxed{?}}$

Keyboard: ⌘ - shift - T
Menu: Templates > Calc > Total Derivative
Rewrite Commands: Eval Derivative, Evaluate
Standard Macro: \der

The Total Derivative template is a binary operator. While both of its operands can be arbitrary expressions, it is customary for the variable of differentiation to be an atom.

The Eval Derivative command under the Simp menu evaluates both total and partial derivatives. In the Total Derivative template, all atoms are treated as if they are dependent on the variable of differentiation. The Evaluate command in the Simp menu will evaluate only one derivative while applying the chain rule. The Move Right and Move Left commands recognize derivatives to the extent that a term can be dragged out of a derivative only if it does not depend on the variable of differentiation.

Partial derivative $\frac{\partial ?}{\partial \boxed{?}}$

Keyboard: ⌘ - T
Menu: Templates > Calc > Partial Derivative
Rewrite Commands: Eval Derivative, Evaluate
Standard Macro: \derp

The Partial Derivative template in the Calc submenu of the Templates menu is a binary operator. While both operands can be arbitrary expressions, it is customary for the variable of differentiation to be an atom.

The Eval Derivative command under the Simp menu evaluates both total and partial derivatives. In a partial derivative template, all the atoms other than the variable of differentiation are treated as constants. The Evaluate command in the

Simp menu will evaluate one level of a derivative without applying the chain rule. Click algebra recognizes derivatives in that you can drag a term out of a derivative only if it doesn't depend on the variable of differentiation.

Nth Total Derivative $\frac{d^{?}\boxed{?}}{d?}$

Keyboard: None
Menu: Templates > Calc > Nth Total Derivative
Rewrite Commands: Evaluate
Standard Macro: \nth

The Nth Total Derivative operator is inserted from the Calc submenu of the Templates menu. The Evaluate command in the Simp menu expands the Nth Total Derivative operator, provided that "N" is an integer between 0 and 20. If "N" is not an integer, the Evaluate command will leave the expression untouched.

For example, $\frac{d^2u}{dx}$ is expanded to $\frac{d\ du}{dx\,dx}$ by the Evaluate command because "N" is an integer between 0 and 20 but $\frac{d^nu}{dx}$ is not affected by the same command.

Nth Partial Derivative $\frac{\partial^{?}\boxed{?}}{\partial ?}$

Keyboard: None
Menu: Templates > Calc > Nth Partial Derivative
Rewrite Commands: Evaluate
Standard Macro: \nthp

The Nth Partial Derivative operator is inserted using the Calc submenu of the Templates menu. The Evaluate command will expand the Nth Partial Derivative operator provided that "N" is an integer between 0 and 20. If "N" is not an integer, the Evaluate command will leave the expression untouched. The Evaluate command will expand both Nth Partial and Nth Total derivatives if the "N" is an integer.

So $\frac{\partial^2 u}{\partial x}$ is expanded to $\frac{\partial\ \partial u}{\partial x \partial x}$ by the Evaluate command because "N" is an integer between 0 and 20 but

$\frac{\partial^n u}{\partial x}$ is not affected by the Eval Derivative command.

Definite Integral $\int_{?}^{?} \boxed{?}\, d?$

Keyboard: ⌘ - shift - 1
Menu: Templates > Calc > Definite Integral
Rewrite Commands: Eval Integrals
Standard Macro: \defint

The Definite Integral operator is inserted using either the Calc submenu of the Templates menu or the ⌘ - shift - 1 key combination. The standard macro \defint can be used to insert the template.

The Eval Integrals command evaluates this template by rewriting it as an indefinite integral evaluated between its limits.

Indefinite Integral $\int \boxed{?}\, d?$

Keyboard: opt - 1
Menu: Templates > Calc > Indefinite Integral
Rewrite Commands: Eval Integrals
Standard Macro: \int

The Indefinite Integral operator is inserted using either the Calc submenu of the Templates menu or using the opt - 1 key combination. The standard macro \int can be used to insert the template as well.

The Eval Integrals command in the Simp menu evaluates polynomial indefinite integrals. Other indefinite integrals must be evaluated using Milo's pattern matcher.

Path integral $\int_{?}\boxed{?}$

Keyboard: None
Menu: Templates > Calc > Path Integral
Rewrite Commands: Eval Integrals
Standard Macro: \path

The Path Integral operator is inserted using either the Calc submenu of the Templates menu or the standard macro \path.

Loop integral $\oint\boxed{?}d?$

Keyboard: None
Menu: Templates > Calc > Loop Integral
Rewrite Commands: Eval Integrals
Standard Macro: \loop

The Loop Integral operator is inserted using either the Calc submenu of the Templates menu or the standard macro \loop.

Comma $,\boxed{?}$

Keyboard: None
Menu: Templates > Calc > Comma
Rewrite Commands: None
Standard Macro: \comma

The Comma is inserted using the Calc submenu of the Templates menu. It has no built-in meaning and is not evaluated by Milo. It is purely a syntactic unary operator.

Covariant Derivative $;\boxed{?}$

Keyboard: None
Menu: Templates > Calc > Covariant Derivative
Rewrite Commands: None
Standard Macro: \;

The Covariant Derivative template is inserted using the Calc submenu of the Templates menu. It has no built-in meaning and is not evaluated by Milo. It is purely a syntactic unary operator.

Differential Operator $\frac{d}{d\boxed{?}}$

Keyboard: None
Menu: Templates > Calc > Differential Operator
Rewrite Commands: None
Standard Macro: \totalop

The Differential template looks the same on the screen as a product. To tell them apart, try to select the "d" with the mouse. If the "d" is selectable by itself, it is an atom and part of a product. If the "d" is not selectable by itself, it is part of the differential operator.

Partial Operator $\frac{\partial}{\partial\boxed{?}}$

Keyboard: None
Menu: Templates > Calc > Partial Operator
Rewrite Commands: None
Standard Macro: \partialop

Differential $d\boxed{?}$

Keyboard: None
Menu: Templates > Calc > Differential
Rewrite Commands: None
Standard Macro: \d

Variation $\delta\boxed{?}$

Keyboard: None
Menu: Templates > Calc > Variation
Rewrite Commands: None
Standard Macro: \var

Delta $\Delta\boxed{?}$

Keyboard: None
Menu: Templates > Calc > Delta
Rewrite Commands: None
Standard Macro: \delta, \Delta

Bra $\langle \boxed{?} |$

Keyboard: None
Menu: Templates > Calc > Bra
Rewrite Commands: None
Standard Macro: \bra

Ket $| \boxed{?} \rangle$

Keyboard: None
Menu: Templates > Calc > Ket
Rewrite Commands: None
Standard Macro: \ket

Bracket $\langle \boxed{?} | ? \rangle$

Keyboard: None
Menu: Templates > Calc > Bracket
Rewrite Commands: None
Standard Macro: \bracket

All of the above seven operators have no meaning anywhere else in Milo and are included only for purposes of display.

Diacritical Marks

Atoms can have diacritical marks associated with them. The diacritical marks that Milo provides include primes, dots, tildes, bars and vector symbols. The Simplify command may not recognize some diacritical marks. For example $\bar{a} + a$ incorrectly simplifies to $2a$.

Vector $\vec{a}$

Keyboard: $\boxed{\text{opt}}$ - $\boxed{\text{Y}}$
Menu: Diac > Vector
Rewrite Commands: None
Standard Macro: None

Typing $\boxed{\text{opt}}$ - $\boxed{\text{Y}}$ on a selection will put vector symbols over all the atoms in the selection, and remove any vector symbols already there. An atom can have either a tilde, a bar, or a vector symbol over it, but cannot have any combination of them at the same time.

Bar ā

Keyboard: _
Menu: Diac > Bar
Rewrite Commands: None
Standard Macro: None

Typing underscore _ after an atom puts a bar over it. Typing _ again removes the bar. An atom can have either a tilde, a bar, or a vector symbol over it, but cannot have any combination of them.

Hat â

Keyboard: opt - 6
Menu: Diac > Hat
Rewrite Commands: None
Standard Macro: None

Typing opt - 6 after an atom puts a hat over it. Typing opt - 6 again removes the hat. An atom can have either a tilde, a bar, or a hat symbol over it, but cannot have any combination of them.

Dot ȧ

Keyboard: .
Menu: Diac > Dot
Rewrite Commands: None
Standard Macro: None

Typing a period . after an atom puts a dot on top of it. Typing the . key again makes it a double dot, then a triple dot, and then removes the dots.

Prime a'

Keyboard: ` (backquote)
Menu: Diac > Prime
Rewrite Commands: None
Standard Macro: None

Typing the backquote key ` after an atom puts a prime after it. Typing the ` key twice makes it a double prime, and typing three times makes it a triple prime. The fourth consecutive backquote typed will remove all the prime markings from the atom.

Tilde ã

Keyboard: ~
Menu: Diac > Tilde
Rewrite Commands: None
Standard Macro: None

Typing a tilde ~ after an atom puts a tilde over it. Typing ~ again removes it. An atom can have either a tilde, a bar, or a vector symbol over it, but cannot have any combination of the three.

Sub Tilde a̰

Keyboard: opt - shift - ’
Menu: Diac > Sub Tilde
Rewrite Commands: Simplify, Multiply Out
Standard Macro: None

Typing opt - shift - ’ after an atom puts a tilde under that atom. Typing opt - shift - ’ again removes the subtilde. An atom can have a tilde or a bar beneath it, but not both.

Sub Box ?

Keyboard: opt - shift - −
Menu: Diac > Sub Box
Rewrite Commands: Simplify, Multiply Out
Standard Macro: None

Typing opt - shift - − after an atom puts a bar under it. Typing opt - shift - − again removes the bar. An atom can have a tilde or a bar beneath it, but not both at the same time.

Dummy a

Keyboard: ⌘ - opt - D
Menu: Diac > Dummy
Rewrite Commands: None
Standard Macro: None

The Dummy template is used to designate wildcard variables in user-defined rewrite rules. Dummy variables are shown in bold face and will take on the value of any expression when using the Apply Rule command in the Rules menu. To create a dummy variable, use the $\boxed{⌘}$ - $\boxed{\text{opt}}$ - $\boxed{\text{D}}$ key sequence or the Dummy command in the Diac menu on a selected atom or with the insertion point directly after the atom.

Other Input Commands

Assignment $\boxed{?} \leftarrow ?$

Keyboard: $\boxed{:}$
Menu: Templates > More > Assignment
Rewrite Commands: Apply Rule
Standard Macro: None

The assignment operator is read "gets." The Pattern Matcher replaces all occurrences of the left side of the assignment operator with the right side. For example in the assignment $a \leftarrow b$, all occurrences of "a" will be replaced with "b" by the Apply Rule command. The pattern matcher treats this the same as a = b, in fact the equals operator can be used interchangeably with the assignment operator in this context.

Next expression ? $\boxed{?}$

Keyboard: $\boxed{;}$
Menu: Templates > More > Next Expression
Rewrite Commands: None
Standard Macro: None

The Next Expression operator is an n-ary operator used to build more than one expression on one line.

Choice Function $\binom{\boxed{?}}{?}$

Keyboard: None
Menu: Templates > More > Choice Function
Rewrite Commands: Evaluate
Standard Macro: \choice

The Choice operator is the standard combinatorics choice function. The Evaluate command under Simp menu evaluates the choice function in the following way:

$$\binom{n}{m} = \frac{n!}{m!(n-m)!}$$

Commutator [?, ?]

Keyboard: None
Menu: Templates > More > Commutator
Rewrite Commands: None
Standard Macro: \comm

The Commutator template is a binary operator with no built-in meaning anywhere else in Milo.

Anticommutator {?, ?}

Keyboard: None
Menu: Templates > More > Anticommutator
Rewrite Commands: None
Standard Macro: \acomm

The Anticommutator is a binary operator with no built-in meaning anywhere else in Milo.

Function ?(?)

Keyboard: opt - 9
Menu: Templates > More > Function
Rewrite Commands: None
Standard Macro: None

The Function operator describes a functional dependence in Milo and has no built-in meaning anywhere else in the program. Typing , after the parameter will add new parameters.

Dagger $\boxed{?}^{+}$

Keyboard: opt - shift - T
Menu: Templates > More > Dagger
Rewrite Commands:None
Standard Macro: None

The dagger operator is purely syntactic unary operator with no built-in meaning anywhere else in Milo.

Chapter 5 Rewriting Expressions

What this chapter contains

All the simplify commands contained in the

- Simp menu
- Tfm menu

This chapter describes in detail every Milo command relating to the manipulation of expressions. Each entry in the chapter describes a Milo command and information about how to use that command.

Most of the items in the Simp menu simplify an expression by making it smaller and more elegant. Most of the items under the Tfm menu transform an expression expanding it into a larger, more explicit form. The Rules menu, discussed in the following chapter is used to define custom rewrite rules.

Milo's ability to graph expressions is not discussed in this chapter, but reserved for the chapter entitled Graphing Expressions.

The format of this chapter

Each rewrite command in this chapter is listed as it appears in Milo's menus. Relevant information such as keyboard equivalents and modifier keys that alter the function of the command are listed directly below the command name. The location of the command in the menus is shown using the greater-than symbol > to denote submenus and finally to point to the command name. This is followed by a complete description of the command.

Manipulating Expressions in Milo

To manipulate an expression in Milo, first select it and then choose a command from the menus. In some cases a rewrite command operates on a range larger than the selected expression, but in all cases the commands affect only expressions on the current line.

Complicated expressions can be quickly massaged by hand using correctness-preserving transformations. These are commands that transform an expression into an equivalent form. The most useful correctness-preserving transformations in Milo are the move-right and move-left commands (See the Tips and Techniques chapter). These commands allow algebraic transformations to be done in a WYSIWYG fashion, by moving terms around in an equation with the mouse. Milo applies the appropriate rules to maintain the correctness of the expression.

The Simp Menu

Simplify

Keyboard: ⌘ - A
Menu: Simp > Simplify
Modifier Keys: opt shift

The Simplify command listed in the Simp menu performs algebraic simplification. It lies at the heart of Milo's power. The Simplify command will perform the following tasks:

- 32-bit integer arithmetic
- Group terms in a product
- Group terms in a sum
- Cancel common terms in the numerator and denominator of a fraction
- Recognize the iota (opt - shift - I) as the complex number i.

In all of the following examples, selecting the left hand side of an equality and choosing the Simplify command from the Simp menu will produce the right hand side of the equality.

Integer Arithmatic

$0x=0$ $1+1=2$ $3!=6$ $4!=24$ $\mathit{log}\ e=1$ $\mathit{log}\ 1=0$

$a+a=2a$

Grouping Terms

$(3a+1)+(b^2-a+2)=b^2+2a+3$ $e^x e^y=e^{x+y}$

Canceling in a fraction using the Simplify command

$\frac{x^a}{x^b}=x^{-b+a}$ $\frac{2a^4b^3abc}{4a^3bac^2}=\frac{ab^3}{2c}$

Interpreting the complex i

$$\sqrt{-1}=\imath \qquad \imath^{2}=-1 \qquad \imath^{23}=-\imath \qquad \sqrt{-ab}=\imath\sqrt{a}\sqrt{b}$$

The opt - Simplify command

The Simplify command automatically rewrites $a\frac{x}{b}$ as $\frac{ax}{b}$. In some situations this is undesirable. The distributive rule is not applied to an expression if the opt key is held down when choosing the Simplify command.

In the following example, opt - Simplify will not rewrite the expression while Simplify will combine the fractions:

opt - Simplify: $\frac{a+1}{b}\frac{c}{d+1}=\frac{a+1}{b}\frac{c}{d+1}$

Simplify: $\frac{a+1}{b}\frac{c}{d+1}=\frac{c\left(a+1\right)}{b\left(d+1\right)}$

In this line Simplify will not group terms as is shown, but opt - Simplify will:

Option-Simplify: $1\frac{meter}{sec}+3\frac{meter}{sec}=4\frac{meter}{sec}$

Simplify: $1\frac{meter}{sec}+3\frac{meter}{sec}=\frac{meter}{sec}+\frac{3\,meter}{sec}$

The shift - Simplify

With the shift key held down, Simplify first extends the selection to be the whole line, then performs a standard simplify operation.

Limitations of the Simplify command

All integer arithmetic is done in 32-bits with the Simplify command. No overflow checking is performed so errors do occur with large integers as shown below. Watch carefully for this and use floating point numbers if you suspect a potential overflow.

$$(2^2)(3^3)(5^5)(7^7) = -1227111740 \qquad 2^{31} = --2147483648 \qquad 2^{32} = 0$$

Milo's simplifier also makes errors simplifying $\sqrt{x^2}$. Milo simplifies this to "x" when the result should be $|x|$.

The simplifier also makes errors when it sees "0/x," replacing it with zero for all "x," so 0/0 gets replaced by 0. Again, you need to watch for these restrictions.

Multiply Out

Keyboard: ⌘ - W
Menu: Simp > Multiply Out
Modifier Keys: opt

The Multiply Out command in the Simp menu does everything that the Simplify command does, but also distributes multiplication over addition. This command goes through the entire selected range and wherever a sum is a term of a product, it multiplies out the product. For example:

$$(a+b)(a-b) = a^2 - b^2$$

$$(x^2+x+1)(x-1) = x^3 - 1$$

If a sum is raised to a positive integer power less than 20, the Multiply Out command will multiply out that power. That is, $(a+b)^2$ becomes $a^2+2ab+b^2$. Applying the Multiply Out command to

$$(a-b)^4 = a^4 - 4ba^3 + 6a^2b^2 - 4ab^3 + b^4$$

we get this:

$$(a+1)^{10} = a^{10} + 10a^9 + 45a^8 + 120a^7 + 210a^6 + 252a^5 + 210a^4 + 120a^3 + 45a^2 + 10a + 1$$

opt- Multiply Out

If the opt key is pressed when choosing the Multiply Out command, Milo will not pass through the the entire selected range, but will stop multiplying out terms after applying its rewrite rules once.

To illustrate this, take the expression a(b+c(d+e)). Multiply Out rewrites this as ab+acd+ace, while opt - Multiply Out rewrites it as ab+ac(d+e).

Add Fractions

Keyboard: ⌘-R
Menu: Simp > Add Fractions
Modifier Keys: None

The Add Fractions command in the Simp menu does everything that the Simplify command does, but also adds fractions by putting them over a common denominator. For example

$\frac{a}{b}+\frac{c}{d}$ becomes $\frac{ad+bc}{bd}$

The Simplify command will not group terms of a fraction. The Add Fractions command must be used explicitly to do this:

Simplify: $\frac{c}{2}+\frac{c}{3}=\frac{c}{2}+\frac{c}{3}$

Add Fractions: $\frac{c}{2}+\frac{c}{3}=\frac{5c}{6}$

Add Fractions will put only the first two terms of a sum over a common denominator. It must be used repeatedly to simplify complicated fractions:

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{y+x}{xy}+\frac{1}{z} \qquad \frac{y+x}{xy}+\frac{1}{z}=\frac{z(y+x)+xy}{xyz}$$

$$\frac{1}{a}-\frac{1}{b}=\frac{b-a}{ab} \qquad \frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$$

$$a+\frac{b}{c}=\frac{ac+b}{c}$$

Order Sum

Keyboard: ⌘ - opt - shift - 0

Menu: Simp > Order Sum

Modifier Keys: shift

The Order Sum command orders polynomials in decreasing powers of a variable. To use it select the variable with respect to which you want the polynomial ordered, then issue the Order Sum command. Milo will look for the first sum above the variable and order that sum in decreasing powers of the selected variable. In the following example, selecting the variable "x" in the expression on the left of the equal sign and choosing the Order Sum command will produce the expression on the right:

$$\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{24}+1+\frac{x^5}{5!}+x=\frac{x^5}{5!}+\frac{x^4}{24}+\frac{x^3}{6}+\frac{x^2}{2}+x+1$$

shift - Order Sum

Holding down the shift key when choosing the Order Sum command will order the polynomial in increasing powers of the selected variable:

$$\frac{x^5}{5!}+\frac{x^4}{24}+\frac{x^3}{6}+\frac{x^2}{2}+x+1=1+x+\left(\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{24}+\frac{x^5}{5!}\right)$$

Number Crunch

Keyboard: ⌘ - K
Menu: Simp > Number Crunch
Modifier Keys: None

The Number Crunch command first changes all integers to floating point numbers in an expression and then numerically evaluates the expression. All arguments to the Number Crunch command must be able to be converted into floating point numbers. In the following examples, the left side of the equal sign is number crunched to produce the right side.

$$1=1. \qquad \tan 0=0. \qquad \sin\frac{\pi}{2}=1.$$

Number Crunch replaces "π" and "e" with floating point numbers unless the "e" is the base of an exponent, or part of a string.

$$\pi=3.14159 \qquad e=2.71828 \qquad e^{3}=20.0855$$

Floating point calculation is not exact. Sin π should number crunch to exactly 0:

$$\sin\pi=4.33681\times10^{-19}$$

Number Crunch crunches big ugly expressions in one step:

$$\frac{\dfrac{1}{\sin 1.5\cos 1.5}+3.-\dfrac{1.5}{1.5^{2}+1}}{\sqrt{1.5^{2}+1}}e^{2.25}\tan 1.5=1240.17$$

Number Crunch will display the error message NaN, meaning "not a number" for illegal operations, and INF, for infinity, when the program overflows.

NaN	Not a Number	Illegal operation performed
INF	Infinity	Overflow detected

$$\frac{1}{0}-\frac{1}{0}=-\text{NaN} \qquad 0\frac{1}{0}=\text{NaN} \qquad \frac{0}{0}=\text{NaN} \qquad \frac{1}{0}=\text{INF}$$

Move Right

Keyboard: opt - .
Menu: Simp > Move Right
Modifier Keys: shift

Move Left

Keyboard: opt - ,
Menu: Simp > Move Left
Modifier Keys: shift

These commands as their names suggest are used to move terms of an expressions right or left. The intent is to bring algebraic simplification to an intuitive level.

The Move Left and Right, listed in the Simp menu, are catch-all commands. They will apply different rules depending on the context. If the selection is a term in the middle of a product, these commands will commute that term with one of its adjacent terms.

Whatever the context, Milo will try to apply a correctness-preserving algebraic rule to move the selection in the desired direction. If correctness cannot be preserved, Milo will not move the term.

The Move Right and Move Left commands can be executed from the Simp menu, their command-keys, or holding down the ⌘ key and clicking to the right or left of an expression.

Command-clicking (holding the ⌘ key down while clicking the mouse) to the right or left of a selected range is exactly the same as selecting the Move Right or Move Left menu item. **Command-clicking above or below an expression works in the same way to move expressions from the numerator to the denominator of a fraction or vice-versa, while preserving the correctness of the expression.**

shift - Move Right
shift - Move Left

With the shift key held down, the Move Right and Move Left commands will attempt to move a term all the way to the right or left side of an expression.

opt - Move Right
opt - Move Left

With the opt key held down, the Move Right and Move Left commands will attempt to distribute a selected range into the adjacent term.

Examples

In the following examples, selecting the bold faced "x" and choosing Move Left (either with the mouse, from the menu, or from the keyboard as ⌘-,, or by command-clicking to the left of the selection range) will produce the expression on the right. In most cases either Move Right, or opt - Move Right will do the inverse operation to get back to the original expression.

$ab\mathbf{x}cd$	→	$a\mathbf{x}bcd$	$a=\mathbf{x}+b$	→	$a-\mathbf{x}=b$
$a=\mathbf{x}b$	→	$a\frac{1}{\mathbf{x}}=b$	$\mathbf{x}a+\mathbf{x}b$	→	$\mathbf{x}(a+b)$
$\sqrt{\mathbf{x}a}$	→	$\sqrt{a}\sqrt{\mathbf{x}}$	$(\mathbf{x}ab)$	→	$\mathbf{x}(ab)$
$(\mathbf{x}b)^n$	→	$\mathbf{x}^n b^n$	$\mathbf{x}^n$	→	$\mathbf{x}\mathbf{x}^{n-1}$
$\frac{\mathbf{x}a}{b}$	→	$\mathbf{x}\frac{a}{b}$	$\frac{a}{\mathbf{x}b}$	→	$\frac{1}{\mathbf{x}}\frac{a}{b}$
$\frac{\partial}{\partial t}\mathbf{x}ab$	→	$\mathbf{x}\frac{\partial}{\partial t}ab$			
$\int\mathbf{x}abdv$	→	$\mathbf{x}\int abdv$			

The following equation may be solved for "x" by moving the "by" term across the equal sign, then moving the coefficient "a" across the equal sign:

$$ax+by=c \qquad ax=-by+c \qquad xa=-by+c \qquad x=\frac{1}{a}(-by+c)$$

Factor

Keyboard: ⌘ - F
Menu: Simp > Factor
Modifier Keys: opt shift

The Factor command in the Simp menu factors a term out of a product. The Factor command does not do polynomial factorization. If you want to do polynomial factorization, you must first guess what one of the factors might be and then use the Long Division command to test it. To use the Factor command, select the desired term and execute the command. The factoring process is equivalent to repeatedly moving the term left. For example, factoring one of the "a's" out of ab+ca+d yields a(b+c+d/a).

$$ax + bx \qquad x\left(a+b\right)$$

$$a^2 + a + 1 \qquad aa^1 + a + 1 \qquad a\left(a+1+\frac{1}{a}\right)$$

opt - Factor

With the opt key down, Factor will only factor the selected term out of terms which contain it. This is useful for grouping terms. For example, with an "a" selected in the expression ab+ca+d, the opt - Factor command produces d+(b+c)a.

The following example was generated by first selecting the "x^2" term and doing an opt - Factor, then selecting an "x" and doing another opt - Factor.

$$ax^2 + bx + c + dx^2 + ex + d \qquad \left(bx + c + ex + d\right) + \left(a+d\right)x^2$$

$$c + d + x\left(b+e\right) + x^2\left(a+d\right)$$

The following was done by selecting the i and doing opt - Factor.

$$a + b\imath + c + d\imath \qquad \left(a+c\right) + \left(b+d\right)\imath$$

Solve For

Keyboard: ⌘ - Y
Menu: Simp > Solve For
Modifier Keys: None

The Solve For command in the Simp menu isolates an expression on one side of the equal sign. This is accomplished by applying the inverse operations of every term on the same side to both sides of the equality. For example, to solve for "x" in ax+by=c, select the "x" and choose Solve For.

Note that in the expression $x^2+2x+1=0$, selecting an "x" and choosing Solve For does not combine the two "x's" in the line, so it has merely isolated the term.

The following examples were generated by selecting the "x" in the expression on the left and choosing Solve For.

$$ax+by=c \quad \rightarrow \quad x=\frac{c-by}{a}$$

$$\boxed{x}^2+x+1=0 \quad \rightarrow \quad x=\sqrt{-\boxed{x}-1}$$

$$x^2+x+1=0 \quad \rightarrow \quad x=-x^2-1$$

$$y=\mathit{log}\ x \quad \rightarrow \quad e^y=x$$

$$\sqrt{y}=x \quad \rightarrow \quad y=x^2$$

$$R\mathit{sin}^2x+by+c\sqrt{z}=0 \quad \rightarrow \quad x=\mathit{asin}\ \frac{\sqrt{-by-c\sqrt{z}}}{\sqrt{R}}$$

Distribute

Keyboard: ⌘ - J
Menu: Simp > Distribute
Modifier Keys: None

The Distribute command, listed in the Simp menu, is a catch-all command that does different things depending on the context. The following lists the situations where the Distribute command is most useful.

- Distributing division over addition
- Dealing with radicals and powers
- Distributing multiplication over addition

One of the more common uses of this command is to distribute division over addition.

After selecting the expression $\frac{a+b}{2}$ the Distribute command produces $\frac{a}{2}+\frac{b}{2}$.

In this case, the Add Fractions command in the Simp menu performs the transformation in the opposite direction.

The Distribute command can be useful when dealing with radicals and powers. $b\sqrt{a}$ distributes to $\sqrt{b^2 a}$, ba^2 distributes to $\left(\sqrt{b}\,a\right)^2$. The Distribute command does the same things in a fraction as it does in a product, i.e. $\frac{\sqrt{a}}{b}$ becomes $\sqrt{\frac{a}{b^2}}$ and $\frac{a^2}{b}$ becomes $\left(\frac{a}{\sqrt{b}}\right)^2$.

In the above cases, the Simplify command will perform the inverse transformation of the Distribute command.

The Distribute command also will distribute multiplication over addition. Selecting the expression $a(b+c)$ and executing the Distribute command produces $ab+ac$.

In general, if the selection range is a product then the Distribute command does the same things as $\boxed{\texttt{opt}}$ - Move Right or $\boxed{\texttt{opt}}$ - Move Left.

$\frac{a+b}{2}$	→	$\frac{a}{2}+\frac{b}{2}$	$a\sqrt{b}$	→	$\sqrt{a^2 b}$
$a^2 b$	→	$\left(a\sqrt{b}\right)^2$	$a^2 b^2$	→	$(ab)^2$
$\frac{\sqrt{a}}{b}$	→	$\sqrt{\frac{a}{b^2}}$	$\frac{\sqrt{a}}{\sqrt{b}}$	→	$\sqrt{\frac{a}{b}}$

$$\frac{a^b}{b} \rightarrow \left(\frac{a}{b^{1/b}}\right)^b$$

$$a+b+c \rightarrow -(-a-b-c)$$

$$a(b+c) \rightarrow ab+ac$$

$$-a-b-c \rightarrow -(a+b+c)$$

$$\sum_{n=a}^{b} u+v+w \rightarrow \sum_{n=a}^{b} u+ \sum_{n=a}^{b} v+ \sum_{n=a}^{b} w$$

Distribute Over Equality

Keyboard: ⌘-=
Menu: Simp > Distribute Over Equality
Modifier Keys: None

The Distribute Over Equality command, located in the Simp menu, is used to perform the same operation to both sides of an equal sign.

In the following examples, selecting the expression on the left and choosing the Distribute Over Equality command will produce the expression on the right.

$$(a=b)+1 \rightarrow a+1=b+1$$

$$(a=b)^2 \rightarrow a^2=b^2$$

$$\sqrt{(a=b)} \rightarrow \sqrt{a}=\sqrt{b}$$

$$-(a=b) \rightarrow -a=-b$$

$\sqrt{(x^2-a=b)+a} \rightarrow \sqrt{(x^2-a)+a}=\sqrt{b+a}$ which simplifies to $x=\sqrt{b+a}$

Evaluate

Keyboard: ⌘ - E
Menu: Simp > Evaluate
Modifier Keys: None

The Evaluate command is a catch-all command that does many unrelated things depending on the selected range.

Evaluating a Derivative
The Evaluate command will evaluate the top level of a total or partial derivative. (This is the same as the opt-Eval Derivative command)

$$\frac{d}{dt}e^{t^2} \qquad e^{t^2}\frac{d}{dt}t^2$$

Evaluating a Nth derivative
If a derivative is raised to a positive integer less than 20, the Evaluate command will rewrite a derivative as n first derivatives.

$$\frac{\partial^4}{\partial t}f(t) \qquad \frac{\partial\ \partial\ \partial\ \partial}{\partial t\,\partial t\,\partial t\,\partial t}f(t)$$

Evaluating a Positive integer power
If an expression is raised to a power less than 20, the Evaluate command will rewrite it as a product.

$$x^4 \qquad xxxx$$

Evaluating a Factorial
The Evaluate command will compute the factorial of any integer less than 13.

$5! = 120$

$6! = 720$

$12! = 479001600$

Evaluating a Absolute Value

The Evaluate command will compute the determinant of a 1x1 or 2x2 matrix.

$$\left|\begin{bmatrix}x\end{bmatrix}\right| = x$$

$$\left|\begin{bmatrix}a & b\\ c & d\end{bmatrix}\right| = ad-bc$$

Evaluating a Summation or Product

The Evaluate command will pull a term out of a sum or product. This is the same as the Eval Sums or Products command discussed later in this chapter.

Evaluating a Substitution

The Evaluate command will evaluate a substitution. (This is the same as ⌘ - \).

$$t\,e^{t}\Big|_{t=x^2} \qquad x^2 e^{x^2}$$

Evaluating a Eval Between

The Evaluate command will rewrite the Eval Between template as a difference between the endpoints.

$$f(x)\Big|_{x=a}^{b} \qquad f(x)\Big|_{x=b} - f(x)\Big|_{x=a}$$

Evaluating a Log

The Evaluate command will rewrite a log of a product or power as the sum of logarithms.

$$\mathit{log}\,(abc) \qquad \mathit{log}\,a+\mathit{log}\,b+\mathit{log}\,c$$

$$\mathit{log}\,x^{n} \qquad n\,\mathit{log}\,x$$

Evaluating a Log Base b

If an expression is of the form $log_b x$, the Evaluate command will rewrite the expression in terms of the natural log.

$$log_b x \qquad\qquad \frac{log\ x}{log\ b}$$

Evaluating a Choice function

The choice function is rewritten by the Evaluate command in terms of factorials.

$$\binom{n}{m} \qquad\qquad \frac{n!}{m!(n-m)!}$$

Eval Integrals

Keyboard: ⌘ - I
Menu: Simp > Eval Integrals
Modifier Keys: None

The Eval Integrals command can only evaluate polynomial integrals. It rewrites definite integrals as indefinite integrals evaluated between the limits. To integrate a polynomial it is necessary to use this command several times and you can see the steps Milo follows in evaluating the integral.

Applying the Eval Integrals command four times to the first expression below produces the other three.

$$\int_0^a x^2+x+1\,dx \qquad\qquad \int x^2+x+1\,dx\Big|_{x=0}^{a}$$

$$\int x^2 dx+\int x\,dx+\int 1\,dx\Big|_{x=0}^{a} \qquad\qquad \frac{x^3}{3}+\frac{x^2}{2}+x\Bigg|_{x=0}^{a}$$

Eval Derivatives

Keyboard: ⌘ - B
Menu: Simp > Evaluate Derivatives
Modifier Keys: opt

The Eval Derivatives command evaluates derivatives, applying the chain rule when necessary in complex expressions. When evaluating partial derivatives, all dependencies must be explicitly written. When evaluating total derivatives, the

Eval Derivatives command treats all atoms as if they depended on the variable of differentiation. This command does not evaluate nth derivatives, which must be expanded using the Evaluate command first.

opt - Eval Derivatives

With the opt key down, the Eval Derivatives command only evaluates the top level of each derivative and does not evaluate each derivative generated by the chain rule.

In the examples below, the Eval Derivatives command is applied to the first expression to produce the second. The opt Eval Derivatives command produces the third expression.

$$\frac{\partial}{\partial x}abx^2 \quad \to \quad 2abx \quad \to \quad a\frac{\partial}{\partial x}bx^2+bx^2\frac{\partial a}{\partial x}$$

$$\frac{d}{dx}abx^2 \quad \to \quad a\left(2bx+x^2\frac{db}{dx}\right)+bx^2\frac{da}{dx} \quad \to \quad a\frac{d}{dx}bx^2+bx^2\frac{da}{dx}$$

$$\frac{\partial}{\partial x}\mathit{tan}\ x \quad \to \quad \mathit{sec}\ ^2x \quad \to \quad \mathit{sec}\ ^2x\frac{\partial x}{\partial x}$$

$$\frac{\partial}{\partial x}e^{x^{x^x}} \quad \to \quad x^{x^x}e^{x^{x^x}}\left(x^x\left(\mathit{log}\ x+1\right)\mathit{log}\ x+x^{x-1}\right) \quad \to \quad e^{x^{x^x}}\frac{\partial}{\partial x}x^{x^x}$$

$$\frac{\partial}{\partial x}\left(\mathit{sin}\ ^2x+\mathit{cos}\ ^2x\right) \quad \to \quad 0 \quad \to \quad \frac{\partial}{\partial x}\mathit{sin}\ ^2x+\frac{\partial}{\partial x}\mathit{cos}\ ^2x$$

$$\frac{\partial}{\partial x}\sqrt{x^2-1} \quad \to \quad x\left(x^2-1\right)^{-1/2} \quad \to \quad \sqrt{x^2-1}\left(\mathit{log}\ \left(x^2-1\right)\frac{\partial}{\partial x}\left(\frac{1}{2}\right)+\frac{\frac{\partial}{\partial x}\left(x^2-1\right)}{2\left(x^2-1\right)}\right)$$

Eval Sums or Products

Keyboard: ⌘ - P
Menu: Simp > Eval Sums or Products
Modifier Keys: shift

The Eval Sums or Products command pulls a term out of a sum or product and increments the appropriate index.

shift - Eval Sums or Products

If the shift key is held down when the Eval Sums or Products command is invoked and both sum or product indices are positive and their difference is less than 100, this command will write out all terms in the sum or product.

In the following examples, applying Eval Sums or Products to the first expression creates the second expression. The third expression is the result of applying shift - Eval Sums or Products.

$$\sum_{n=0}^{5} a_n \rightarrow a_0 + \sum_{n=1}^{5} a_n x^n \rightarrow a_0 + x a_1 + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5$$

$$\sum_{n=0}^{\infty} -1^n \frac{x^{2n+1}}{(2n+1)!} \rightarrow x + \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$\prod_{i=0}^{3} x+i \rightarrow x \prod_{i=1}^{3} x+i \rightarrow x(x+1)(x+2)(x+3)$$

$$\prod_{i=1}^{5} i \rightarrow \prod_{i=2}^{5} i \rightarrow 1(2)3(4)5$$

$$\sum_{n=1}^{10} n \rightarrow \sum_{n=2}^{10} n+1 \rightarrow 1+2+3+4+5+6+7+8+9+10$$

Eval Substitutions

Keyboard: ⌘ - \
Menu: Simp > Eval Substitutions
Modifier Keys: None

The Eval Substitutions command searches through the selection range and wherever it finds a substitution template, the substitution will be lexically performed and the result will automatically be simplified.

$$a\Big|_{a=b} \quad \rightarrow \quad b$$

$$\left.\frac{e^x \sin x}{x^2+1}\right|_{x=0.5} \quad \rightarrow \quad \frac{e^{0.5} \sin 0.5}{0.5^2+1}$$

$$f(x)\Big|_{x=b} - f(x)\Big|_{x=a} \quad \rightarrow \quad f(b)-f(a)$$

The Tfm Menu

Rectangular

Keyboard: None
Menu: Tfm > Rectangular

Cylindrical

Keyboard: None
Menu: Tfm > Cylindrical

Spherical

Keyboard: None
Menu: Tfm > Spherical

These commands expand the Div, Grad, and Curl operators in the appropriate coordinate system. The divergence and curl will be expanded only if their argument is a three component matrix.

The following are the gradient, divergence, and curl:

$$\nabla f \qquad \nabla \bullet \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \qquad \nabla \times \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

in rectangular:

$$\begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{bmatrix} \qquad \frac{\partial}{\partial x}E_1 + \frac{\partial}{\partial y}E_2 + \frac{\partial}{\partial z}E_3 \qquad \begin{bmatrix} \frac{\partial}{\partial y}E_3 - \frac{\partial}{\partial z}E_2 \\ \frac{\partial}{\partial z}E_1 - \frac{\partial}{\partial x}E_3 \\ \frac{\partial}{\partial x}E_2 - \frac{\partial}{\partial y}E_1 \end{bmatrix}$$

cylindrical:

$$\begin{bmatrix} \frac{\partial f}{\partial r} \\ \frac{1}{r}\frac{\partial f}{\partial \theta} \\ \frac{\partial f}{\partial z} \end{bmatrix} \qquad \frac{1}{r}\frac{\partial}{\partial r}r\,E_1 + \frac{1}{r}\frac{\partial}{\partial \theta}E_2 + \frac{\partial}{\partial z}E_3 \qquad \begin{bmatrix} \frac{1}{r}\frac{\partial}{\partial \theta}E_3 - \frac{\partial}{\partial z}E_2 \\ \frac{\partial}{\partial z}E_1 - \frac{\partial}{\partial r}E_3 \\ \frac{1}{r}\left(\frac{\partial}{\partial r}r\,E_2 - \frac{\partial}{\partial \theta}E_1\right) \end{bmatrix}$$

and spherical coordinates:

$$\begin{bmatrix} \frac{\partial f}{\partial r} \\ \frac{1}{r}\frac{\partial f}{\partial \theta} \\ \frac{1}{r\,\mathit{sin}\ \theta}\frac{\partial f}{\partial \Phi} \end{bmatrix} \qquad \frac{1}{r^2}\frac{\partial}{\partial r}r^2 E_1+\frac{1}{r\,\mathit{sin}\ \theta}\frac{\partial}{\partial \theta}E_2\,\mathit{sin}\ \theta+\frac{1}{r\,\mathit{sin}\ \theta}\frac{\partial}{\partial \Phi}E_3$$

$$\begin{bmatrix} \frac{1}{r\,\mathit{sin}\ \theta}\left(\frac{\partial}{\partial \theta}E_3\,\mathit{sin}\ \theta-\frac{\partial}{\partial \Phi}E_2\right) \\ \frac{1}{r\,\mathit{sin}\ \theta}\left(\frac{\partial}{\partial \Phi}E_1-\mathit{sin}\ \theta\frac{\partial}{\partial r}r\,E_3\right) \\ \frac{1}{r}\left(\frac{\partial}{\partial r}r\,E_2-\frac{\partial}{\partial \theta}E_1\right) \end{bmatrix}$$

The Rectangular, Cylindrical and Spherical commands will not expand the Laplacian template. To find the laplacian, evaluate the divergence of the gradient in the desired coordinate systems as follows:

In rectangular:

$$\nabla^2 U=\nabla\bullet\nabla\ U=\nabla\bullet\begin{bmatrix} \frac{\partial U}{\partial x} \\ \frac{\partial U}{\partial y} \\ \frac{\partial U}{\partial z} \end{bmatrix}=\frac{\partial\ \partial U}{\partial x\partial x}+\frac{\partial\ \partial U}{\partial y\partial y}+\frac{\partial\ \partial U}{\partial z\partial z}$$

cylindrical:

$$\nabla^2 U = \nabla \bullet \nabla\, U = \nabla \bullet \begin{bmatrix} \frac{\partial U}{\partial r} \\ \frac{1}{r}\frac{\partial U}{\partial \theta} \\ \frac{\partial U}{\partial z} \end{bmatrix} = \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial U}{\partial r} + \frac{1}{r}\frac{\partial}{\partial \theta}\frac{1}{r}\frac{\partial U}{\partial \theta} + \frac{\partial}{\partial z}\frac{\partial U}{\partial z}$$

and spherical:

$$\nabla^2 U = \nabla \bullet \nabla\, U = \nabla \bullet \begin{bmatrix} \frac{\partial U}{\partial r} \\ \frac{1}{r}\frac{\partial U}{\partial \theta} \\ \frac{1}{r\, sin\, \theta}\frac{\partial U}{\partial \Phi} \end{bmatrix} = \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial U}{\partial r} + \frac{1}{r\, sin\, \theta}\frac{\partial}{\partial \theta}\left(\frac{1}{r}\frac{\partial U}{\partial \theta}\right) sin\, \theta + \frac{1}{r\, sin\, \theta}\frac{\partial}{\partial \Phi}\frac{1}{r\, sin\, \theta}\frac{\partial U}{\partial \Phi}$$

More Examples of Coordinate Systems

Evaluating the following in rectangular coordinates and then evaluating the derivatives by choosing Eval Derivatives gives

$$\nabla \left(x^2 + y^2\right) = \begin{bmatrix} \frac{\partial}{\partial x}\left(x^2 + y^2\right) \\ \frac{\partial}{\partial y}\left(x^2 + y^2\right) \\ \frac{\partial}{\partial z}\left(x^2 + y^2\right) \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \\ 0 \end{bmatrix}$$

Evaluating the laplacian of r^2 in spherical and evaluating the derivates yields 6.

$$\nabla \bullet \nabla r^2 = \nabla \bullet \begin{bmatrix} \frac{\partial}{\partial r} r^2 \\ \frac{1}{r}\frac{\partial}{\partial \theta} r^2 \\ \frac{1}{r\, sin\, \theta}\frac{\partial}{\partial \Phi} r^2 \end{bmatrix} = 6$$

This result can be checked in rectangular coordinates knowing that $r^2=x^2+y^2+z^2$.

$$\nabla \bullet \nabla \left(x^2+y^2+z^2\right) = \nabla \bullet \begin{bmatrix} \frac{\partial}{\partial x}\left(x^2+y^2+z^2\right) \\ \frac{\partial}{\partial y}\left(x^2+y^2+z^2\right) \\ \frac{\partial}{\partial z}\left(x^2+y^2+z^2\right) \end{bmatrix} = \nabla \bullet \begin{bmatrix} 2x \\ 2y \\ 2z \end{bmatrix} = \frac{\partial}{\partial x}2x + \frac{\partial}{\partial y}2y + \frac{\partial}{\partial z}2z = 6$$

Long Division

Keyboard: ⌘ - opt - L
Menu: Tfm > Long Division
Modifier Keys: None

The Long Division command performs polynomial long division. To use it, select a variable in a fraction of two polynomials.

In each of the following lines, selecting the blackened "x" and choosing the Long Division command produces the next expression.

$$\frac{\mathbf{x}^2+2x+1}{x+1} \rightarrow x+\frac{1\mathbf{x}+1}{x+1} \rightarrow x+1$$

$$\frac{\mathbf{x}^2+2x+2}{x+1} \rightarrow x+\frac{1\mathbf{x}+2}{x+1} \rightarrow x+\left(\frac{1}{x+1}+1\right)$$

$$\frac{\mathbf{x}^3-a^3}{x-a} \rightarrow x^2+\frac{-a^3+a\mathbf{x}^2}{x-a} \rightarrow x^2+\left(ax+\frac{-a^3+a^2\mathbf{x}}{x-a}\right) \rightarrow x^2+\left(ax+a^2\right)$$

$$\frac{\mathbf{x}+2}{x+1} \rightarrow \frac{1}{x+1}+1$$

Change Coordinates

Keyboard: None
Menu: Tfm > Change Coordinates
Modifier Keys: None

The Change Coordinates command changes vectors between spherical, rectangular, and cylindrical coordinate systems. The selection must be a 1x3 or a 3x1 matrix. Choosing the Change Coordinates command brings up a dialog box that prompts for the transformation to apply, upon which Milo applies one of following four rewrite rules.

Rectangular to spherical:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ goes to } \begin{bmatrix} \sqrt{x^2+y^2+z^2} \\ atan\,\frac{\sqrt{x^2+y^2}}{z} \\ atan\,\frac{y}{x} \end{bmatrix}$$

Rectangular to cylindrical:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ goes to } \begin{bmatrix} \sqrt{x^2+y^2} \\ atan\,\frac{y}{x} \\ z \end{bmatrix}$$

Spherical to rectangular:

$\begin{bmatrix} r \\ \theta \\ \Phi \end{bmatrix}$ in spherical goes to $\begin{bmatrix} r\,sin\,\theta\,cos\,\Phi \\ r\,sin\,\theta\,sin\,\Phi \\ r\,cos\,\theta \end{bmatrix}$ in rectangular.

Cylindrical to rectangular:

$\begin{bmatrix} r \\ \theta \\ z \end{bmatrix}$ goes to $\begin{bmatrix} r\,cos\,\theta \\ r\,sin\,\theta \\ z \end{bmatrix}$ in rectangular.

Trig → Base Trig

Keyboard: None
Menu: Tfm > Trig -> Base Trig

This command rewrites trigonometric expressions in terms of sin and cos, or sinh and cosh, by applying the following rewrite rules.

$$tan\ x \rightarrow \frac{sin\ x}{cos\ x}$$

$$tanh\ x \rightarrow \frac{sinh\ x}{cosh\ x}$$

$$csc\ x \rightarrow \frac{1}{sin\ x}$$

$$sech\ x \rightarrow \frac{1}{cosh\ x}$$

$$cot\ x \rightarrow \frac{cos\ x}{sin\ x}$$

$$csch\ x \rightarrow \frac{1}{sinh\ x}$$

$$sec\ x \rightarrow \frac{1}{cos\ x}$$

1) $\coth x \rightarrow \dfrac{\cosh x}{\sinh x}$

2) $\cos^2 x \sec x \tan x \rightarrow \cos^2 x \dfrac{1}{\cos x}\dfrac{\sin x}{\cos x}$ which simplifies to *sin*x

3) $1-\sec^2 x+\tan^2 x \rightarrow 1-\left(\dfrac{1}{\cos x}\right)^2+\left(\dfrac{\sin x}{\cos x}\right)^2$ upon which the Add Fractions command produces

$\dfrac{\sin^2 x+\cos^2 x-1}{\cos^2 x}$ Here the user has to replace $\sin^2 x + \cos^2 x$ by 1 to get

$\dfrac{1-1}{\cos^2 x}$ which simplifies to 0 using the Simplify command.

Trig → Exponential

Keyboard: ⌘-opt-shift-X
Menu: Tfm > Trig -> Exponential

This command rewrites selected trigonometric expressions in terms of exponentials by applying the following rewrite rules.

$$\sin x \rightarrow \frac{e^{\iota x}-e^{-\iota x}}{2\iota}$$

$$\sinh x \rightarrow \frac{e^{x}-e^{-x}}{2}$$

$$\cos x \rightarrow \frac{e^{\iota x}+e^{-\iota x}}{2}$$

$$\cosh x \rightarrow \frac{e^{x}+e^{-x}}{2}$$

$$\tan x \rightarrow -\iota\frac{e^{\iota x}-e^{-\iota x}}{e^{\iota x}+e^{-\iota x}}$$

$$\tanh x \rightarrow \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$$

$$\sin^{2}x+\cos^{2}x \rightarrow \left(\frac{e^{\imath x}-e^{-\imath x}}{2\imath}\right)^{2}+\left(\frac{e^{\imath x}+e^{-\imath x}}{2}\right)^{2}$$

The Add Fractions command produces $\dfrac{-\left(e^{x\imath}-e^{-x\imath}\right)^{2}+\left(e^{x\imath}+e^{-x\imath}\right)^{2}}{4}$

and the Multiply Out command produces $\dfrac{2(1)+2}{4}$, which simplifies to 1.

The same steps show the following identity.

$$\cosh^{2}x-\sinh^{2}x = \left(\frac{e^{x}+e^{-x}}{2}\right)^{2}-\left(\frac{e^{x}-e^{-x}}{2}\right)^{2}=$$

$$\frac{\left(e^{x}+e^{-x}\right)^{2}-\left(e^{x}-e^{-x}\right)^{2}}{4} = \mathbf{1}$$

$2\sin x\cos x \rightarrow 2\dfrac{e^{\imath x}-e^{-\imath x}}{2\imath}\dfrac{e^{\imath x}+e^{-\imath x}}{2}$, which Multiplies Out to

$\dfrac{e^{2x\imath}-\left(e^{-2x\imath}+1\right)+1}{2\imath}$, and Simplifies to $\dfrac{e^{2x\imath}-e^{-2x\imath}}{2\imath}$.

Using the Trig -> Base Trig command on $\sin(2x)$ produces $\dfrac{e^{\imath(2x)}-e^{-\imath(2x)}}{2\imath}$, simplifying to

$\dfrac{e^{2x\imath}-e^{-2x\imath}}{2\imath}$. This demonstrates the identity 2*sin*x*cos*x = *sin*2x

Taylor Expand

Keyboard: ⌘ - opt - shift - T
Menu: Tfm > Taylor Expand

The Taylor Expand command rewrites *sin*, *cos*, and e^x as an infinite series by applying the following rewrite rules.

The examples below illustrate the effect of applying the Taylor Expand command followed by the Eval Sums or Products command, or ⌘ - P, twice or four times.

$$\sin x \rightarrow \sum_{n=0}^{\infty} -1^n \frac{x^{2n+1}}{(2n+1)!} \rightarrow x-\frac{x^3}{6}$$

$$\cos x \rightarrow \sum_{n=0}^{\infty} -1^n \frac{x^{2n}}{(2n)!} \rightarrow 1-\frac{x^2}{2}$$

$$e^x \rightarrow \sum_{n=0}^{\infty} \frac{x^n}{n!} \rightarrow 1+x+\frac{x^2}{2}+\frac{x^3}{6}$$

Remove Division

Keyboard: opt - [
Menu: Tfm > Remove Division

Remove Neg Powers

Keyboard: opt -]
Menu: Tfm > Remove Neg Powers

The Remove Division command rewrites all division operators in a selected expression as a multiplication of a negative power. The inverse operation, Remove Neg Powers, rewrites negative powers as a division by a positive power.

These commands undo one another (with the exception of the last example below), so choosing Remove Division from the Tfm menu and applying it to an expression in the first column will rewrite it as the second column, and choosing the Remove Neg Powers command on an expression in the second column will rewrite it as the first column.

$\frac{a}{b}$	→	ab^{-1}
$\frac{1}{b}$	→	b^{-1}
$\frac{a}{bcd}$	→	$ab^{-1}c^{-1}d^{-1}$
$\frac{1}{e^{x}}$	→	e^{-1x}
e^{-x}	→	e^{-x}

Notice that Milo does not rewrite e^{-x} as $\frac{1}{e^{x}}$ simply because it is more useful in mathematics to leave "e" in the numerator.

Vectors → Matrices

Keyboard: ⌘ - opt - J
Menu: Tfm > Vectors -> Matrices

The Vectors -> Matrices command rewrites atoms with the vector diacritical mark as 1x3 column matrix.

$$\vec{A} \rightarrow \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix}$$

$$\vec{x} \bullet \vec{x} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \bullet \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\vec{A} \bullet \left(\vec{B} \times \vec{C}\right) - \left(\vec{B} \bullet \left(\vec{C} \times \vec{A}\right)\right) \rightarrow \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} \bullet \left(\begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix} \times \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} \right) - \left(\begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix} \bullet \left(\begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} \times \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} \right) \right)$$

Transpose Matrix

Keyboard: ⌘ - opt - T
Menu: Tfm > Transpose Matrix

The Transpose Matrix command transposes a selected n-by-m matrix, rewriting it as an m-by-n matrix.

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} \rightarrow \begin{bmatrix} a & b & c \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \rightarrow \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ & \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

Matrix Algebra

Keyboard: ⌘ - opt - M
Menu: Tfm > Matrix Algebra

The Matrix Algebra command performs matrix multiplication and addition and dot and cross products on any selected matrix.

$$-\begin{bmatrix}a\\b\\c\end{bmatrix} \rightarrow \begin{bmatrix}-a\\-b\\-c\end{bmatrix}$$

$$2\begin{bmatrix}a\\b\\c\end{bmatrix} \rightarrow \begin{bmatrix}2a\\2b\\2c\end{bmatrix}$$

$$\begin{bmatrix}10\\20\\30\end{bmatrix}+\begin{bmatrix}0\\1\\\Psi\end{bmatrix} \rightarrow \begin{bmatrix}10+0\\20+1\\30+\Psi\end{bmatrix}$$, which simplifies to $$\begin{bmatrix}10\\21\\\Psi+30\end{bmatrix}$$

$$\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}\bullet\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} \rightarrow x_1x_1+x_2x_2+x_3x_3$$ which simplifies to $x_1^2+x_2^2+x_3^2$

$\vec{A}\bullet(\vec{B}\times\vec{C})-(\vec{B}\bullet(\vec{C}\times\vec{A}))$ expanding these vectors gives

$$\begin{bmatrix}A_1\\A_2\\A_3\end{bmatrix}\bullet\left(\begin{bmatrix}B_1\\B_2\\B_3\end{bmatrix}\times\begin{bmatrix}C_1\\C_2\\C_3\end{bmatrix}\right)-\left(\begin{bmatrix}B_1\\B_2\\B_3\end{bmatrix}\bullet\left(\begin{bmatrix}C_1\\C_2\\C_3\end{bmatrix}\times\begin{bmatrix}A_1\\A_2\\A_3\end{bmatrix}\right)\right)$$

choosing 'Matrix Algebra' once expands the cross product,

$$\begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} \bullet \left(\begin{bmatrix} B_2 C_3 - B_3 C_2 \\ B_3 C_1 - B_1 C_3 \\ B_1 C_2 - B_2 C_1 \end{bmatrix} \right) - \left(\begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix} \bullet \left(\begin{bmatrix} C_2 A_3 - C_3 A_2 \\ C_3 A_1 - C_1 A_3 \\ C_1 A_2 - C_2 A_1 \end{bmatrix} \right) \right)$$

simplifying and choosing 'Matrix Algebra' again produces

$$\left(A_1 \left(B_2 C_3 - B_3 C_2\right) + A_2 \left(B_3 C_1 - B_1 C_3\right) + A_3 \left(B_1 C_2 - B_2 C_1\right)\right) - \left(B_1 \left(A_3 C_2 - A_2 C_3\right) + B_2 \left(A_1 C_3 - A_3 C_1\right) + B_3 \left(A_2 C_1 - A_1 C_2\right)\right)$$

and after running the Multiply Out command all the terms have canceled leaving 0.

Chapter 6 Using Rules

What this chapter contains

The following commands are discussed in this chapter:

- Enter Rule
- Apply Rule
- Create Menu Item

This chapter offers a guide to building your own rules in Milo. Rules provide direct access to Milo's pattern matcher and the information that follows shows you how to customize the program to your specific needs.

This is also a complete list of all the Tables of Formulas found on the Milo distribution disk, including everything from financial formulas to astronomical quantities.

What is a rule

A rule in Milo is really a transformation rule that specifies how to rewrite expressions. Milo uses a built-in pattern matcher to interpret a rule and apply it to any selected expression.

When you apply a rule to a selected expression, Milo replaces everything in the expression that matches the left hand side of the rule with the right hand side of the rule. For example, when Milo applies the rule $a=b$ to the expression a^3+a^2+a, the program replaces all occurrences of "a" with "b" to produce b^3+b^2+b.

Rule Syntax

Rules are created by typing them into a Milo equation block the way you would enter any other expression. A rule must be in one of the following forms or any combination them:

$$a=b$$

$$a \leftarrow b$$

$$a=b=c$$

The letters "a" and "b" above may be arbitrary expressions. The assignment arrow in the second example above is produced with the colon key $\boxed{:}$, or inserted using the Assignment command in the More submenu of the Templates menu.

Rules also may be taken from the files in the Tables of Formulas folder on the Milo distribution disk.

Rule Sets

Rules may contain multiple assignments and may extend across multiple equation blocks in Milo. Any rule that has more than one assignment is called a rule set. To create a rule set, simply select all the individual assignment expressions. Hold down the opt key before clicking and dragging with the mouse will select across multiple equation blocks. Milo ignores any text blocks that may be in between.

Here are some examples of rule sets that can be selected together:

$$\sin(-\mathbf{x}) = -\sin \mathbf{x} \qquad \cos(-\mathbf{x}) = \cos \mathbf{x} \qquad \tan(-\mathbf{x}) = -\tan \mathbf{x}$$

$$\sin \mathbf{x} = \sqrt{1-\cos^2 \mathbf{x}}$$

$$\cos \mathbf{x} = \sqrt{1-\sin^2 \mathbf{x}}$$

The lines separating equation blocks above can be turned on and off through the Options command in the Edit menu (see the chapter on Other Features).

Dummy variables

Dummy variables are variables in a rule that take on the value of any expression. Dummy variables are interpreted as wild card variables by Milo's pattern matcher. Dummy variables are displayed on the screen in boldface notation to distinguish them from regular atoms. A dummy variable is created by selecting an atom and typing ⌘ - opt - D or by selecting an atom and choosing Dummy from the Diac menu.

The atom "a," the Dummy variable "**a**," and every occurrence of "x" in the rules

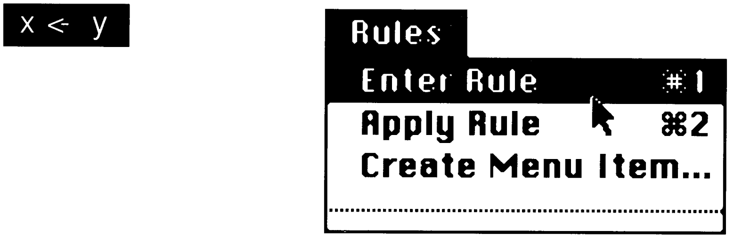
$$\sin(-\mathbf{x}) = -\sin \mathbf{x} \qquad \cos(-\mathbf{x}) = \cos \mathbf{x} \qquad \tan(-\mathbf{x}) = -\tan \mathbf{x}$$

are all a dummy variables.

The Enter Rule command

Once a rule has been built, it can be entered into a buffer as the "active" rule. This is done using the Enter Rule command in the Rule menu, and must be done before a rule can be applied to an expression. Only one rule or rule set can be active at a time, and when a new rule or rule set is entered, the old one is erased.

To enter a rule or rule set, select the desired expression or expressions and choose the Enter Rule command from the Rule menu. For example, to enter a rule that will replace all occurrences of the variable "x" with the variable "y," the setup would look like this:



The Apply Rule command

Once a rule or rule set has been entered, it can be applied to another expression by selecting the expression or expressions that you want to rewrite, and choosing the Apply Rule command from the Rule menu. Milo's pattern matcher will take over and perform all appropriate substitutions.

If you have a rule set in the buffer, the Apply Rule command will search from the top of the rule set to the bottom. As soon as it finds a match, it applies the rule. **This means that the order of the rules in the buffer is important**. The first rule Milo finds that fits is used.

The Create Menu Item command

The Create Menu Item command is a feature that keeps often-used rules at your fingertips. Once you have used the Create Rule command to place an active rule in Milo's buffer, you can store it (or a rule set) as a menu item simply by choosing the Create Menu Item command from the Rule menu.

A menu item may be a user-specified name or the default name, User Item A, User Item B etc..

Milo does not remember menu items or rules once you quit the program, so it is a good idea to keep often-used rewrite rules in a separate Milo file. Many such rules are kept in the folder called Tables of Formulas supplied on the Milo distribution disk. A list of the available files in the folder is included at the end of this chapter.

Adding new templates

Rules in Milo may be used as an effective way of creating new templates. New templates composed of existing Milo primitives may be stored in the Rule menu and applied easily to expressions. For example, applying the rule

$$a \leftarrow \int_{0}^{\infty} a \, dx$$

to any selected expression will wrap the integral template with respect to "x" around the expression.

Tables of Formulas on the Milo distribution disk

The Milo disk contains a folder called Tables of Formulas that includes a wealth of information in the form of rules that may be used with the pattern matcher to solve all sorts of complex problems. The tables included on the disk include:

- Astronomical Quantities
- cgs Physical Constants
- Cubic Equation
- Determinants
- Financial Formulas
- Integral Table
- Integrals - loop and e^x
- Integrals - Rational Fractions
- Integrals - Trig Functions
- Integrals - Definite
- SI Physical Constants
- Units Ratchet

These files are accessed using the Open command in the File menu, and it is worth taking the time to browse through them to familiarize yourself with the formulas available.

Chapter 7 Graphing Expressions

What this chapter contains

- How to create graphs
- Examples of graphing techniques

This chapter explains how to graph expressions in Milo. There are two parts to the chapter, the first part introduces the graphing features and outlines what you can do with graphs in Milo. The second part contains examples of graphing techniques.

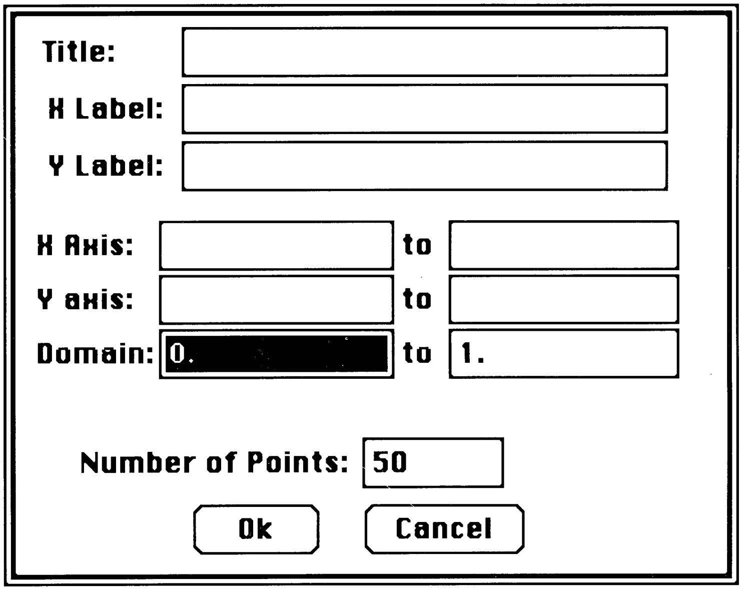
Using Milo to create graphs

Graphs are created in Milo by selecting an expression or expressions and choosing the Graph Expression command from the Edit menu. The keyboard equivalent for the Graph Expression command is ⌘ - G.

Milo will graph parameterized curves and real-valued functions of one variable, and is able to overlay more than one graph in a window at a time. Unlike expressions and text, graphs are built in a separate graphing window and may then be pasted into a Milo document through the Clipboard.

The Graphing Dialog box

When you select the Graph Expression command the following dialog box will appear on the screen:



Title:

X Label:

Y Label:

X Axis: to

Y axis: to

Domain: 0. to 1.

Number of Points: 50

The dialog box allows you to configure the following parameters:

Title

You can name the graphing window by typing a string in this box. What you type here has no effect on the actual graphing, just on the window. The title defaults to empty, which means no title will be displayed for the resulting graph.

X Label

Name the "x" axis. Defaults to empty.

Y Label

Name the "y" axis. Defaults to empty.

X Axis

Allows you to specify the range of the "x" axis. This defaults to empty and normally should be left empty in which case the axis is automatically scaled to show all the points and use the entire graphing window.

Y Axis

Allows you to specify the range of the "y" axis. This defaults to empty and normally should be left empty in which case the axis is automatically scaled to show all the points and use the entire graphing window.

Domain

The points for the graph are generated by substituting floating point numbers between the starting and ending points of the domain for the variable in the expression to be graphed. For example, graphing the expression $\sin x$ over the specified range from 0 to 6.2, will substitute floating point numbers between 0 and 6.2 for "x," and graph the result.

Number of Points

This parameter allows you to choose the number of points to be plotted in the graph. Logically, the more points you choose, the smoother the graph and the longer it will take to compute. In the upper left corner of the graphing window, the points are counted off during computation. Any integer between two and 512 (the number of pixels on a Mac SE and Plus screen) can be used. For example, when graphing the expression $\sin x$ over the specified range from 0 to 6.2, you can specify the number of points to be 50 and Milo will substitute 50 floating point numbers between 0 and 6.2 to obtain the result.

Real-valued functions of one variable

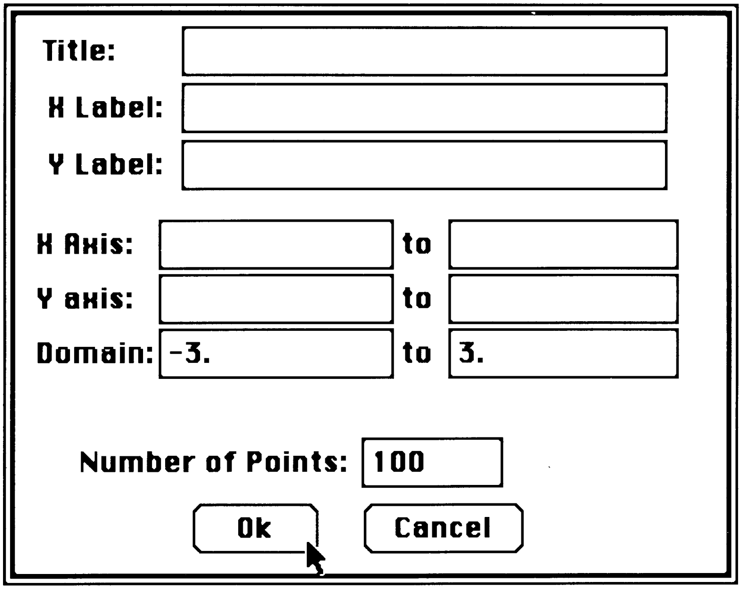
Graphing a single expression is done by selecting the expression and choosing the Graph Expression command in the Edit menu. In order for Milo to graph the expression, the expression must evaluate into a floating-point number. This means that only expressions with one independent variable may be graphed. Milo applies the Number Crunch command automatically when graphing expressions and constants like "e" and "π" are evaluated.

To graph the expression

$$\sin \tan x - \tan \sin x$$

from -3 to 3, select the expression and pick Graph Expression from the Edit menu or simply type ⌘ - G .

Fill in the Graph Expression dialog box to look like this:



Title:

X Label:

Y Label:

X Axis: to

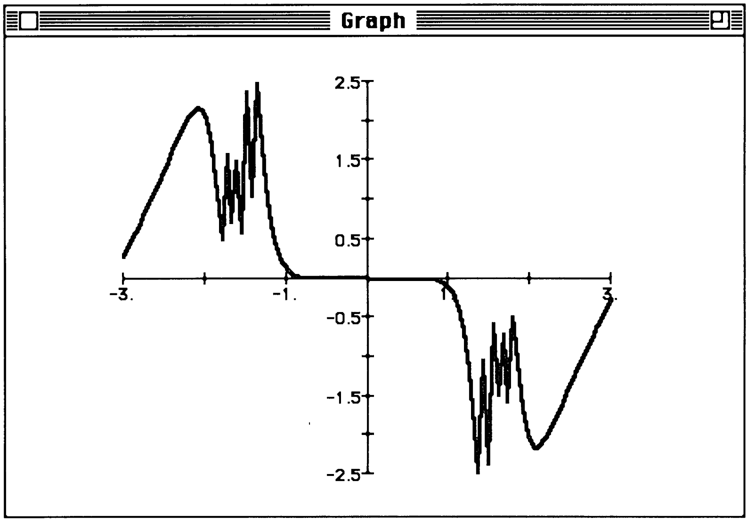
Y axis: to

Domain: -3. to 3.

Number of Points: 100

and click OK. Remember that the tab key jumps the insertion point from cell to cell in the dialog box, just as it cycles the insertion point from ? prompt to ? prompt in a new template.

The resulting graph looks like this:



Graphing parameterized curves

Milo has the ability to plot parameterized curves, where one expression represents the "x" value and another expression represents the "y" value for the graph. In other words,

x = first expression
y = second expression

To graph a parameterized curve, the selected range must be two expressions on the same line, separated by the Next Expression template, available in the Templates menu or by typing the ; key.

Similar to graphing functions of one variable, do not use more than one independent variable in both expressions. If you do, Milo will treat both variables as the same variable and graph the resulting expression.

If you have "1" as the first expression and "1" as the second expression and you plot them as a parametric graph, Milo will plot the point (1,1). Here, no variables are involved.

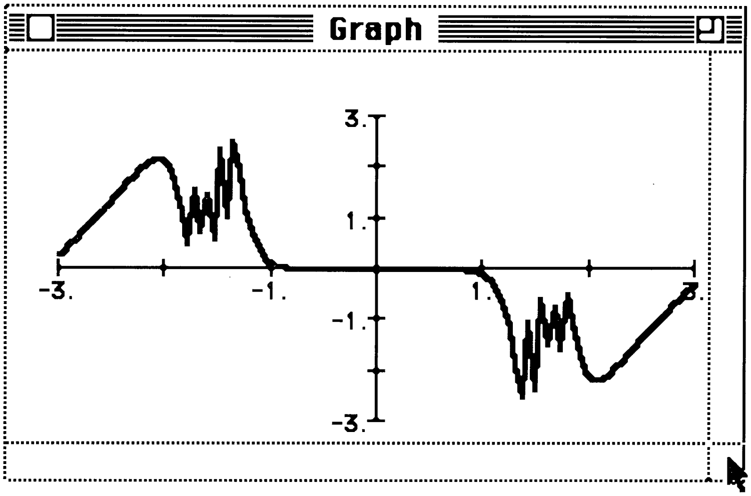
If you have "x" as the first expression and x^2 as the second, it is the same as plotting the non-parametric graph of x^2.

If you have "1" as the first expression and the variable "t" as the second, MIlo will graph a vertical line. Similarly, *sin*t *cos*t graphed between 0 and 2π will produce a circle.

Similarly the expressions *sin*x and *cos*y will be graphed as a circle because Milo treats "x" and "y" as the same variable when graphing parameterized curves.

Resizing the graphing window

The graphing window may be resized by clicking in the lower right corner of the window and dragging, even though the resize icon is not visible. The icon is hidden to enhance the appearance of the graphing window.



Inserting graphs into a Milo document

When the graphing window is active, you may cut it to the Clipboard to transport it into a Milo document. It usually helps to resize the window before transporting it. To paste a graph into a document, make sure the previous expression is not selected.

Milo's graphics block

When a graph is pasted into a Milo document, it is placed in a "graphics block." There are no manipulation commands available in graphics block mode, though the File and Edit menus are available in the menu bar.

Overlaying graphs

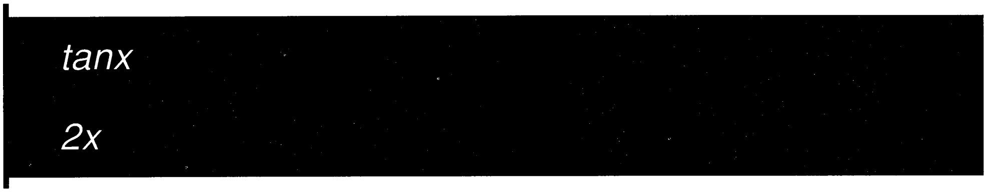
More than one expression in Milo may be graphed simultaneously to overlay one graph on another. This feature is useful, for example, to observe the point or points where graphs intersect.

When graphing the two expressions like *tan* x and 2x , the two expressions must appear on two separate lines in a Milo document (separated by a return). The expressions must be both in separate equation blocks and on two separate lines for them to be overlayed on a graph. To select both expressions at the same time, use the opt - Select command described below.

The opt - Select command

Holding down the opt key when selecting expressions in Milo allows you to select across equation and text blocks to select multiple blocks. The opt - Select command ignores text blocks.

To use the opt - Select command, hold down the opt key and click the mouse near the top expression to be graphed and drag to the bottom expression:

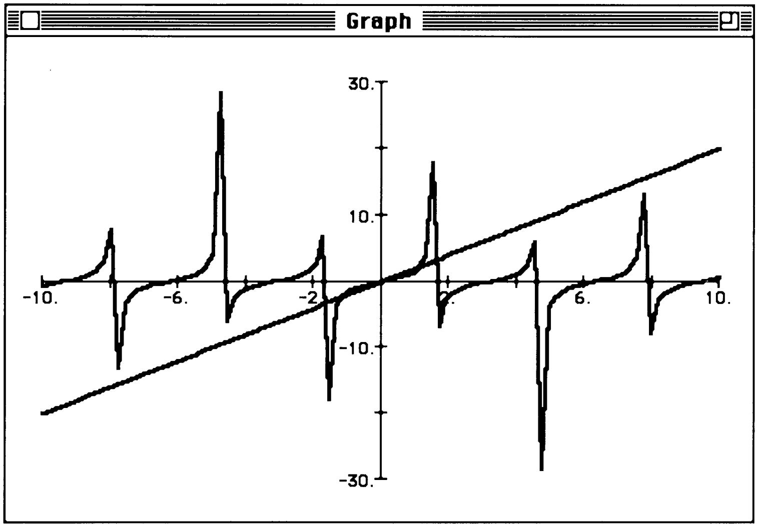


tanx

2x

Graphing two expressions at once

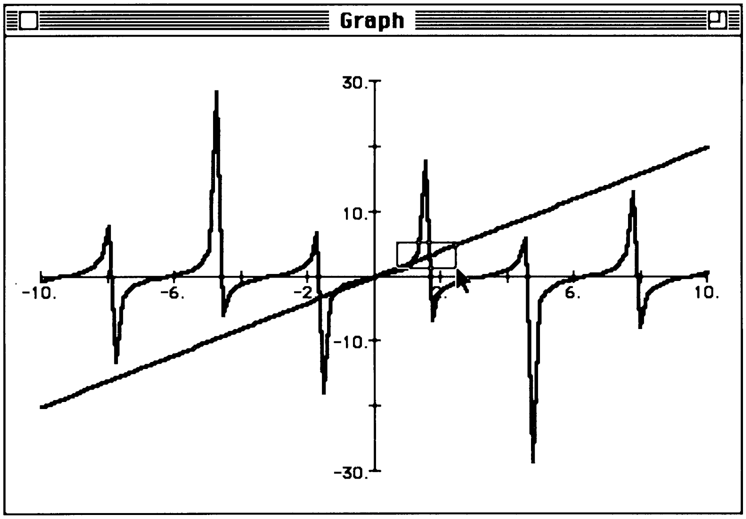
Once you have selected both expressions, use the Graph Expression command from the Edit menu or type ⌘ - G, set the parameters you want in the Graphing dialog box, (for the graph below, the domain was set between -10 and 10, and the number of points was set to 100) and click OK. Here is the graph Milo produces from the above set of expressions:



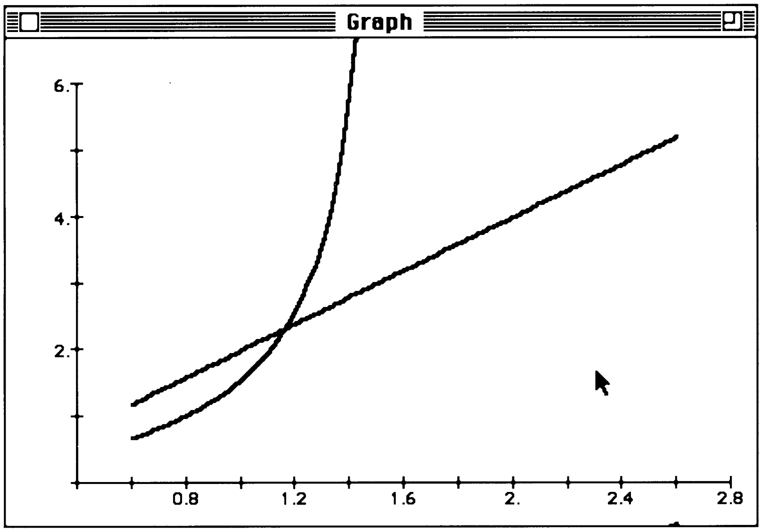
Zooming in on a graph

One of the most useful features of Milo's graphing capability is the zooming feature. To locate maximum and minimum points as well as points of intersection, you can quickly and easily magnify any part of a graph.

To use the zoom feature, you must first graph an expression or expressions. Take, for example the two expressions discussed above `tan x` and `2x`. To zoom in on a particular part of the graph, simply draw a box around the part of the graph you want enlarged, by clicking and dragging with the mouse. Here is an example of a box that will zoom in on the intersection between the two plots:



Here is a picture of the resulting "zoomed" graph:



When you let go of the mouse, the graphing dialog box will appear again, this time containing values for the new graph. Do not change the values for the "x" and "y" axes, since those have been determined by the exact positioning of the zoom box. You can change the number of points for Milo to plot.

opt - Zoom: bypassing the graphing dialog

A faster way to zoom in on a graph is to bypass the graphing dialog box. This is especially useful when you want to zoom in many times on a graph. By holding down the opt key while drawing the zoom box, and keeping it depressed when you let go of the mouse, you can bypass the graphing dialog box and accept the default values for the parameters. The value for the Number Of Points will remain the same as in the original graph, and the "x" and "y" coordinates and Domain parameters will change depending on where you draw the box.

Chapter 8 Other Features in Milo

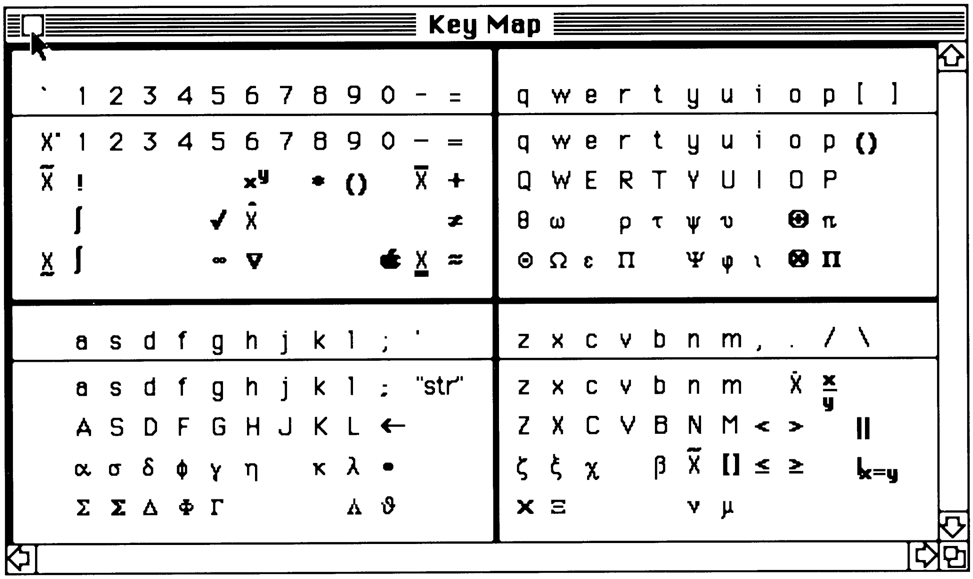
What this chapter contains

This chapter contains information about Milo that doesn't belong anywhere else. These features are found in the Edit menu and include:

- Keymap
- Command Key
- Options
- The Style menu
- Insert Text Block
- Insert Equation Block

Keymap

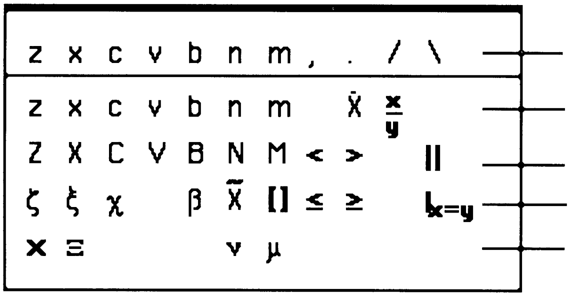
The Keymap command in the Edit menu opens a window that looks like this:



```
` 1 2 3 4 5 6 7 8 9 0 - =        q w e r t y u i o p [ ]
Ẋ 1 2 3 4 5 6 7 8 9 0 − =        q w e r t y u i o p ()
X̄ !         x^y • ()  X̄ +        Q W E R T Y U I O P
  ∫       √ X̂           ≠        θ ω   ρ τ ψ υ   ⊕ π
X̰ ∫       ∞ ∇        X̲ ≈        Θ Ω ε Π   Ψ φ ι ⊗ Π

a s d f g h j k l ; '            z x c v b n m , . / \
a s d f g h j k l ; "str"        z x c v b n m   Ẋ x/y
A S D F G H J K L ←              Z X C V B N M < >   ||
α σ δ ϕ γ η   κ λ •              ζ ξ χ   β X̃ [] ≤ ≥   |x=y
Σ Σ Δ Φ Γ       Λ ϑ              X Ξ       ν μ
```

The first line in each quadrant of the Key Map window is the label on a key on the keyboard. The upper left quadrant represents the top row of keys, and the bottom right quadrant represents the bottom row of keys. The bottom right corner serves as an example below to illustrate how to interpret the Key Map window:



z	x	c	v	b	n	m	,	.	/	\
z	x	c	v	b	n	m		$\dot{X}$	$\frac{x}{y}$	
Z	X	C	V	B	N	M	<	>		‖
ζ	ξ	χ		β	$\tilde{\bar{X}}$	[]	≤	≥		$\lfloor_{x=y}$
×	≡				∨	μ				

Keys exactly as displayed on bottom row of the Mac keyboard.

The atom, operator or diacritical mark the key corresponds to in Milo.

With the shift key depressed.

With the opt key depressed.

With the opt and shift keys depressed.

The second line of each quadrant shows what atom, operator or diacritical mark the key corresponds to in Milo.

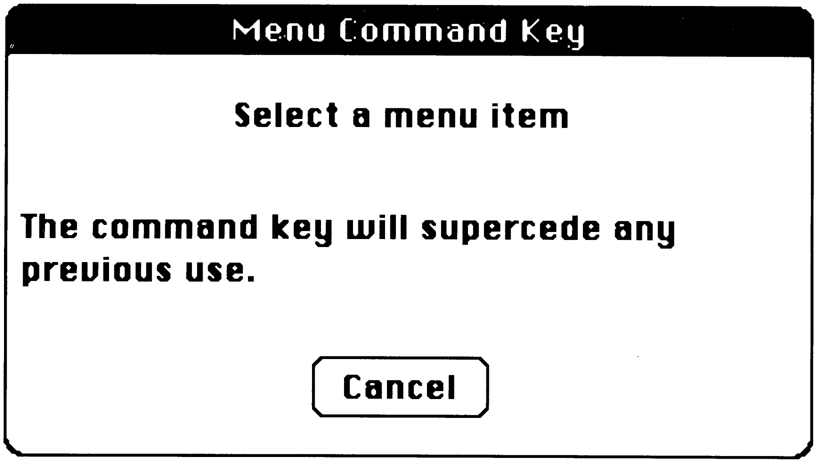
The third line corresponds to holding the shift key down while typing that key.

The fourth line corresponds to holding down the opt key while typing that key.

The last line corresponds to holding both the shift and opt keys down while typing that key.

Command Key

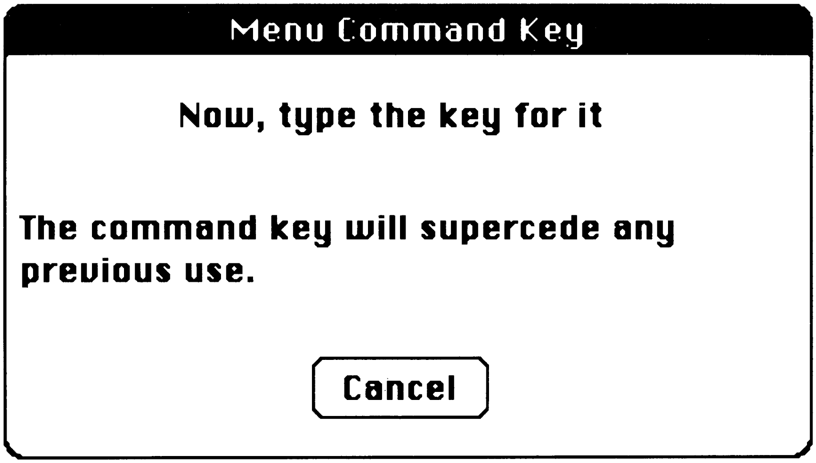
The Command Key command in the Edit menu displays the following dialog box:



Select a menu item

The command key will supercede any previous use.

Selecting a menu item with the mouse when this box is open does not execute that command. Once a command has been selected, the dialog changes:



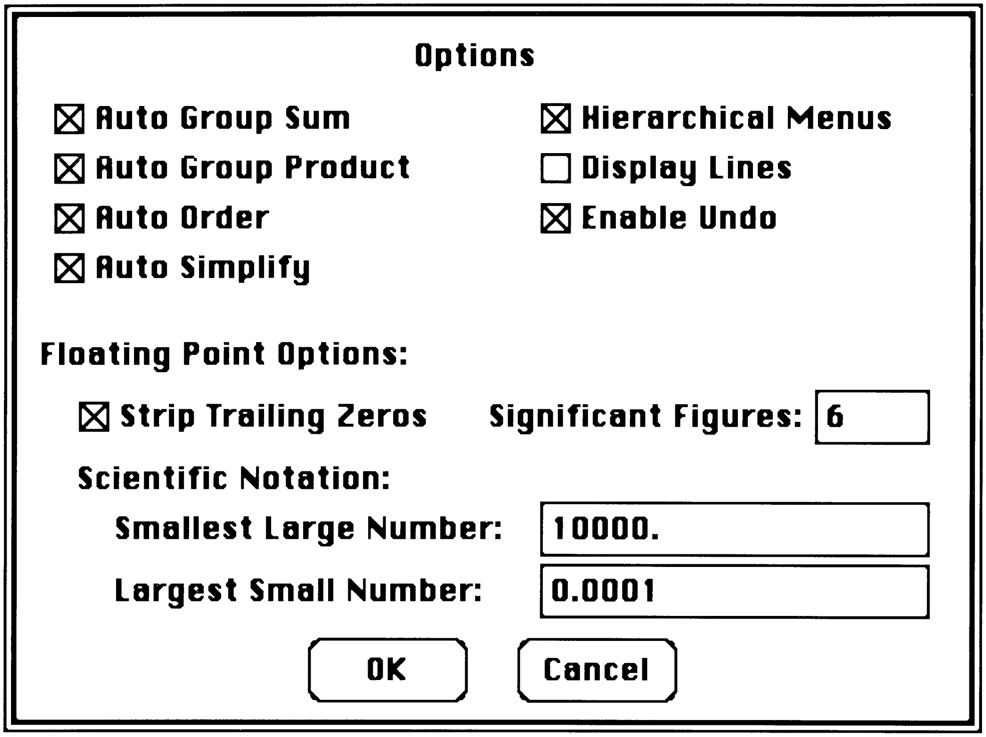
Now, type the key for it

The command key will supercede any previous use.

Now typing a key like ⌘ - A will make ⌘ - A a command-key equivalent for the chosen menu command.

Options

This command brings up the following dialog box:



Options

☒ Auto Group Sum
☒ Auto Group Product
☒ Auto Order
☒ Auto Simplify

☒ Hierarchical Menus
□ Display Lines
☒ Enable Undo

Floating Point Options:

☒ Strip Trailing Zeros

Significant Figures: 6

Scientific Notation:

Smallest Large Number: 10000.

Largest Small Number: 0.0001

Auto Group Sum
Auto Group Product
Auto Order

These settings affect the Simplify command and all commands which use Milo's simplifier. They are normally enabled. Turning off Auto Group Sum will no longer automatically collect terms like "x + x." Turning off Auto Group Product will no longer automatically collect factors like "xx." Turning off Auto Order will no longer automatically order terms in a product alphabetically. These commands may in very rare cases be useful working with non-commutative algebra.

Auto Simplify

Some commands, like Matrix Algebra, automatically invoke the simplifier after they have manipulated an expression. This option can disable the auto simplifying process so that more intermediate steps can be seen.

Hierarchical Menus

The Calculus, Trig, and More menus are by default submenus of the Templates menu. They may, however, be displayed as top-level menus on the menu bar if the Hierarchical Menus option is turned off.

Display Lines

This option causes lines to be drawn horizontally across the screen between each Equation, Text and Graphics block.

Enable Undo

This option enables or disables the Undo feature. When working with very large expressions, the overhead of keeping old copies for Undo may be noticeable. Unless you have a specific reason, it is not recommended that this option be used.

Strip Trailing Zeros

This option controls the display of floating point numbers after the Number Crunch command has been executed. It causes the trailing zeros to be dropped in the string displaying the number.

Significant Figures

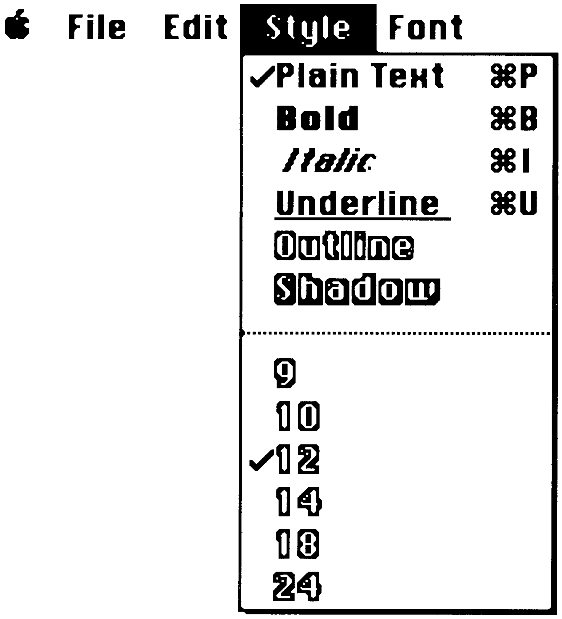
This option controls the number of digits shown when displaying a floating point number. Internal calculations are performed to 19 digits of precision. Numbers are rounded for display and the internal value is not saved. This means that after a result is displayed, no more precision is stored than was displayed. This was done so that results would not be ambiguous. Otherwise, with 1 significant figure set here, 3.0 * 0.3 could be 0.9 if the numbers were as displayed, or it could be 0.99999 if the 0.3 were "really" 0.33333 internally.

Smallest Large Number
Largest Small Number

These settings control when to use scientific notation. Numbers smaller than the largest small and larger than the smallest large number will be automatically displayed in scientific notation.

The Style Menu

When in text mode, Milo displays a Style menu to provide flexible text input:



and the adjacent Font menu offers the choice of fonts installed in the system currently in use.

Insert Text Block

This command, in the Edit menu, creates a new text block below the current block. If the insertion point is in an equation, this is accomplishes the same thing as typing the enter key.

Insert Equation Block

This command, in the Edit menu, creates a new equation block below the current block. If the insertion point is in an equation block, this accomplishes the same thing as typing a return. If the insertion point is in a text block, this is the same as typing the enter key.

Chapter 9 Tips and Techniques

What this chapter contains

Tips are included to help you use Milo effectively in the following areas:

- Quick reference to atoms, templates and rewrite commands
- Fonts
- Selecting expressions
- Menus
- Mouse clicks
- Moving terms around in an expression
- Duplicating expressions
- Floating point numbers, strings and integers
- Special keys
- Keyboard equivalents
- The Command Key option
- The backslash.ini file
- Defining rules
- Graphing

The Tips and Techniques chapter is the place to be if you already consider yourself somewhat of an expert with the Macintosh. This chapter will help you optimize your use of the program.

Quick reference: Atoms

Here is a list of often-used atoms and the fastest way to insert them in a Milo document:

α	Alpha	opt - A
Δ	Delta	opt - shift - D
e	(the constant)	E
∞	Infinity	opt - shift - 5
π	Pi	opt - P

Note that typing shift - E after and integer or a floating point number applies the Scientific Notation template to that number (see the Input Reference chapter).

Quick reference: Templates

$\lvert\boxed{?}\rvert$	Absolute Value	|
$\boxed{?} \leftarrow ?$	Assignment	:
$\int_{?}^{?} \boxed{?}\, d?$	Definite Integral	opt - shift - 1
a	Dummy variable	⌘ - opt - D
$a^{?}$	Exponential	^
$\boxed{?}!$	Factorial	!
$\int \boxed{?}\, d?$	Indefinite Integral	opt - 1
$[\boxed{?}]$	Matrix	opt - M
$? \quad \boxed{?}$	Next expression	;
$\frac{\partial ?}{\partial \boxed{?}}$	Partial derivative	⌘ - T
$\sum_{?=?}^{?} \boxed{?}$	Product	opt - shift - S
$1 \times 10^{?}$	Scientific Notation	shift - E
$\sqrt{\boxed{?}}$	Square Root	opt - 5
$?\big\vert_{\boxed{?}=?}$	Substitution	opt - \
$\prod_{?=?}^{?} \boxed{?}$	Summation	opt - shift - P
$\frac{d?}{d\boxed{?}}$	Total derivative	⌘ - shift - T

Quick reference: Rewrite commands

Add Fractions	⌘ - R
Distribute	⌘ - J
Distribute Over Equality	⌘ - =
Eval Derivatives	⌘ - B
Eval Integrals	⌘ - I
Eval Substitutions	⌘ - \
Eval Sums or Products	⌘ - P
Evaluate	⌘ - E
Factor	⌘ - F
Long Division	⌘ - opt - L
Matrix Algeria	⌘ - opt - M
Move Left	opt - ,
Move Right	opt - .
Multiply Out	⌘ - W
Number Crunch	⌘ - K
Order Sum	⌘ - opt - shift - O
Remove Division	opt - [
Remove Neg Powers	opt -]
Simplify	⌘ - A
Solve For	⌘ - Y
Taylor Expand	⌘ - opt - shift - T
Transpose Matrix	⌘ - opt - T
Vectors -> Matrices	⌘ - opt - J

The best tip on how to use Milo effectively is to use it as a math processor. Although Milo does have the power to manipulate expressions, its real power is in its interface and speed. Here are some tips on how to let you get the most out of the program:

Fonts

If you already have 9- and 12-point Symbol font installed in your system, you can remove those two fonts from the Milo application using Apple's Font/DA Mover to save space. Remember to hold down the option key when clicking on the Open button in the Mover program to be able to open Milo since it is an application, not a system resource file.

If you want to copy and paste expressions created in Milo into a word processing or other application, make sure that application has access to 9- and 12- point Symbol font. If those fonts are not available, display quality will be reduced.

Selecting expressions

To manipulate an expression in Milo, you have to select it. The space bar, the mouse and the up and down arrow keys perform selecting operations:

sp	Even if no expression is previously selected, the space bar will expand a selection range through each valid subexpression until the entire expression is selected.
shift sp	Holding down the shift key when pressing the space bar will de-select an already selected range, the opposite of just using the space bar.
↑	Similar to the space bar, the up arrow key expands a selected range until the entire expression is selected. Part of the expression must be already selected.
↓	The down arrow de-selects an expression similar to the shift-space combination, but some part of the expression must already be selected.
←	The left arrow key cycles the selection range counter-clockwise through an expression selecting each smallest valid subexpression. Some part of the expression must already be selected.
→	The right arrow key cycles the selection range clockwise through an expression in the same way as the left arrow key.
opt sp	Holding down the option key when pressing the space bar performs the same action as the left arrow key. This was designed mainly for use with keyboards that do not have arrow keys.
Mouse	Use the mouse to select all or part of an expression by clicking and dragging. At all times, Milo will only let you select a valid expression or subexpression.

The space bar sp selects an expression starting with the smallest subexpression nearest the insertion point. Press the sp again to select the next largest subexpression. If you keep typing the space bar you will eventually select the entire expression. The ↑ key works the same way except unlike the space bar, the ↑ key requires some subexpression to be already selected.

On older Mac keyboards without arrow keys, ⌘ - Space and opt - Space do the same things as the right and left arrow keys.

Milo will not let you select anything that is not a valid mathematical expression. When you type over a selected range, the range is not replaced but operated on by what you type.

By clicking and dragging with the mouse, you can select any valid subexpression.

Mouse clicks: Moving terms around in an expression

Besides selecting expressions and subexpressions in Milo, the mouse can be used to move terms of an expression right or left relative to other terms. (See the Rewriting Expressions chapter).

Holding down the ⌘ key and clicking to the left or right of a selected term in an expression will move the term in the expression. This is the same as using the Move Term Left and Move Term Right commands in the Simp menu. It does not distribute the term into parenthesis or other subexpressions.

Holding down both the ⌘ and opt keys when clicking to the right or left of a selected term will move that term to the left or right and distribute it into an adjacent subexpression if possible. The same effect can be achieved by holding down the opt key and choosing the Move Term Left or Move Term Right commands from the Simp menu.

Holding down the shift key and issuing a Move Term Left or Move Term Right command will move a selected term to the extreme left or right of an expression. This also can be done by holding down the ⌘ and shift keys and clicking with the mouse to the right or left of a selected term.

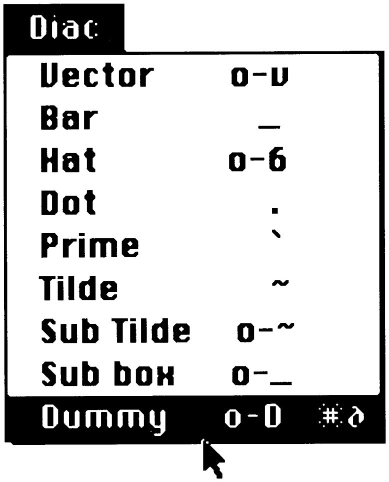
Duplicating expressions

It is a good idea to make copies of your work as you go along so you and anyone else can follow the steps you take to solve a problem. Before you manipulate an expression, select it and copy it to the clipboard with ⌘-C. Then type = and paste it back with ⌘ - Y. Perform your simplification on the right hand side of the equal sign.

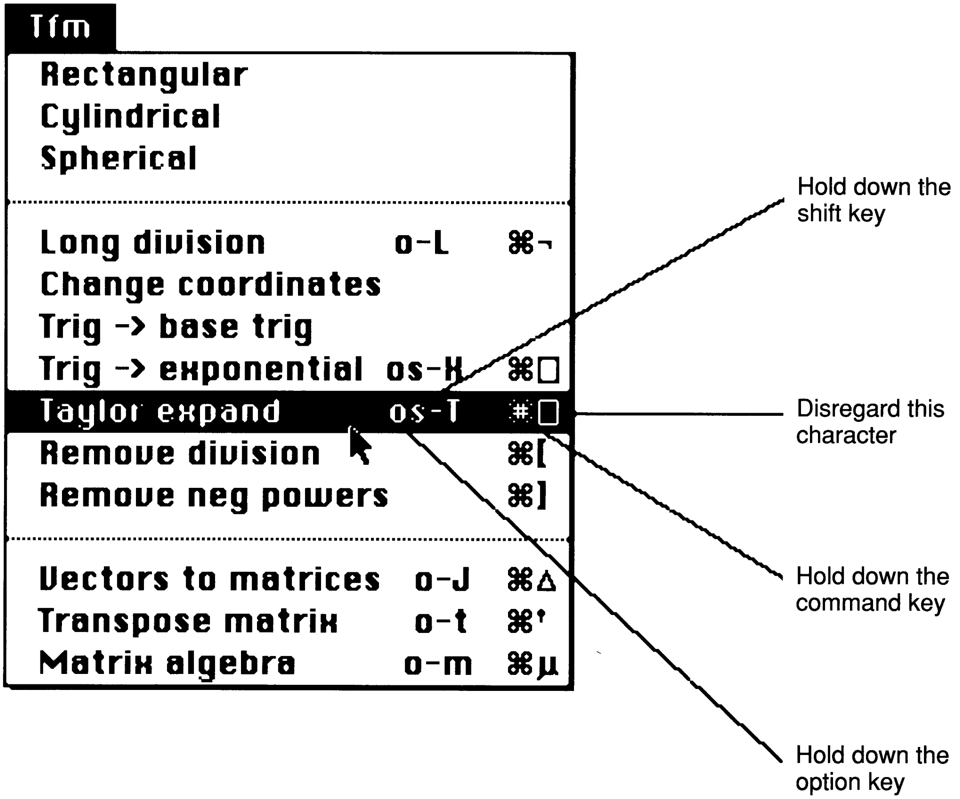
You can also use the Duplicate command in the edit menu that creates a new equation block directly below a selected expression and duplicates it into the new block.

Menus

The keyboard equivalents for some of the menu choices are displayed incorrectly. This occurs when more than one control key is required. For example, the keyboard equivalent for the Dummy template is ⌘-opt-D. This is displayed in the Menus as:



The keyboard equivalent for the Taylor Expand command should be interpreted as follows:



With the shift, ⌘ and opt keys held down, type a T to execute a Taylor Expand command.

Floating point numbers, strings and integers

Typing any number of integers followed immediately by a decimal point creates a string containing those integers. Typing a decimal point with no preceding integers creates a string starting with the number zero followed by a decimal point.

Strings cannot have spaces in them. Use the underscore key _ to connect words together.

Always select a string if you want to operate on it, since it is hard to tell whether the insertion point is inside or outside the string.

Special keys

[;] Next Expression

Use the semicolon key to create more than one expression per line in an equation block. This is used for graphing parametric equations.

[:] Assignment Operator

The assignment operator "←" is produced using the colon key, or accessed through the More submenu of the Templates menu.

[tab]

Use the tab key to cycle through the **?** prompts when filling in a template.

Differences between the Evaluate and the Eval Sums or Products commands.

The Eval Sums or Products command and the Evaluate command are not the same. Evaluate operates on the entire selected expression and evaluates summation or product operators in that expression once. Using the Evaluate command again will have no effect on the expression. The Eval Sums or Products command, on the other hand, keeps performing a substitution each time the command is invoked.

Keyboard equivalents

Keyboard equivalents are pre-defined for many Milo commands and you can create your own by using the "Command Key" option in the Edit menu outlined in this chapter. As you become better acquainted with Milo, using the keyboard instead of the menus will help get things done faster.

The pre-defined keyboard equivalents in Milo, as well as the Macintosh keyboard equivalents, are listed across from the particular selection in the each of the menus.

Command Key option

This option, available in the Edit menu, lets you customize Milo's menus by adding your own command-key equivalents. **Any command-key equivalent you create will override any existing keyboard equivalent with the same name, including the standard Macintosh equivalents like [⌘] - [S] for Save.**

Bypassing the Backslash.ini file

This file can be bypassed on startup by holding down the caps lock key when opening Milo. Holding down the caps lock key tells Milo not to load the file, which speeds up launching the application. Without the Backslash.ini file, Milo's macro facility will not be available.

Defining rules

You can enter rules that are on more than one line in a Milo document. Use opt - click select across multiple lines. When you enter a rule, Milo will ignore any text blocks between expressions, so you can drag right over text blocks to enter rule sets.

Rules are exact lexical replacements. This means that an expression must correspond exactly to a rule or the rule will not be applied.

Rules must be in exact order of precedence. The first rule in a rule set that Milo can apply, will be applied. No other rules in the rule set will be tested.

Once you have created and inserted a menu item using the Create Menu Item command in the Rules menu, you cannot delete it except by exiting the program.

Several files chock full of rules are supplied on the Milo distribution disk. See the chapter called Using Rules for a listing of those files.

Graphing

In a graphing window, straight lines may be drawn by clicking and dragging with the mouse. This allows you to check points on the "x" or "y" axes.

To overlay one graph on another, put two equations in two separate equation blocks and use opt - click-drag to select both lines. Then graph as you normally would.

The Graphing dialog box does not clear or reset the previously-used values, but remembers the values last used. This means if you want to change the settings, you must do so by clearing each field individually.

To insert a graph into a Milo document, cut it and paste it into the document. The graphing window may be resized even though the resize icon is not present. If you want the graph to be small, resize the window and then paste it into the Milo document.

When zooming in on a graph, hold down the opt key as you click and drag the mouse to avoid bringing up the graphing dialog box. This will tell Milo that you accept the default values for the graphing parameters.

When you know a graph is linear, use only two or three points in the domain. This will speed up the graphing process significantly.

Keeping more than one or two graphing windows open at once will slow Milo down so try to keep as few open as possible.

When graphing, make sure you only use one independent parameter. Milo treats all independent parameters as the same variable.

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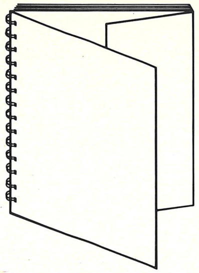

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